Processing-time dependent profit maximization scheduling problems

with applications to star observations

Florian Fontan

under the supervision of Nadia Brauner and Pierre Lemaire

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The star scheduling problem

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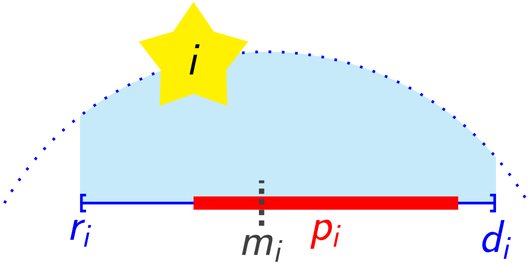

- $[r_i; d_i)$: visibility interval of star i
- p_i: required duration of the observation of star i
- w_i: scientific interest of observing star i

The meridian $m_i \in [r_i, d_i)$ is a mandatory instant of the observation.

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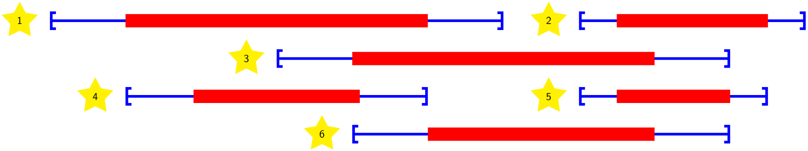
The star scheduling problem

Instance: A set $\mathcal{N}$ of stars; each star $i \in \mathcal{N}$ has an interest w_i, an observation duration p_i and a visibility window $[r_i; d_i)$



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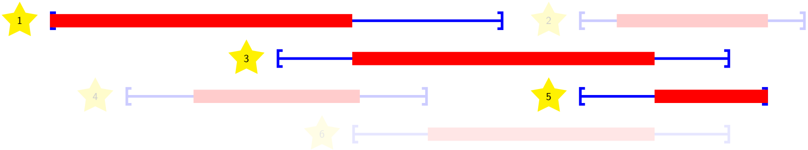
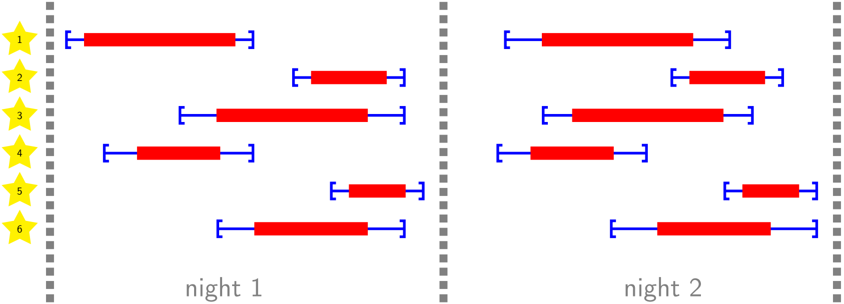


Question: find $\mathcal{N}' \subset \mathcal{N}$ as well as the starting dates of the observations $s_i, \forall i \in \mathcal{N}'$ such that

- for all $i \in \mathcal{N}'$: $[s_i; s_i + p_i) \subset [r_i; d_i)$
- for all $(i_1, i_2) \in \mathcal{N}'^2$: $[s_{i_1}; s_{i_1} + p_{i_1}) \cap [s_{i_2}; s_{i_2} + p_{i_2}) = \emptyset$
- $\sum_{i \in \mathcal{N}'} w_i$ is maximized

The star scheduling problem

Instance: a set $\mathcal{M}$ of nights, a set $\mathcal{N}$ of stars; each star $i \in \mathcal{N}$ has an interest w_i, an observation duration p_i^j and a visibility window $[r_i^j; d_i^j)$, dependent of the night j of the observation.



The practical instances include around 800 stars for a 6 months planification.

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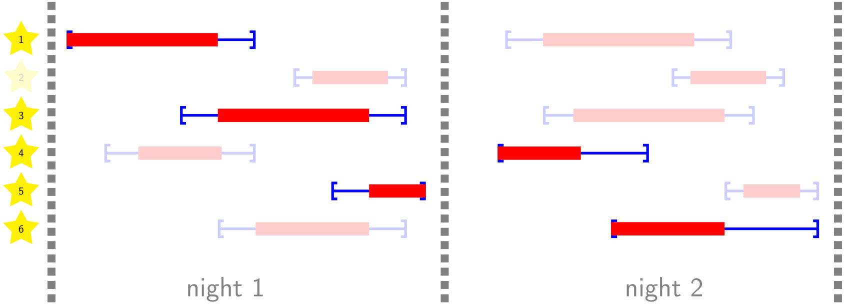


The practical instances include around 800 stars for a 6 months planification.

The star scheduling problem

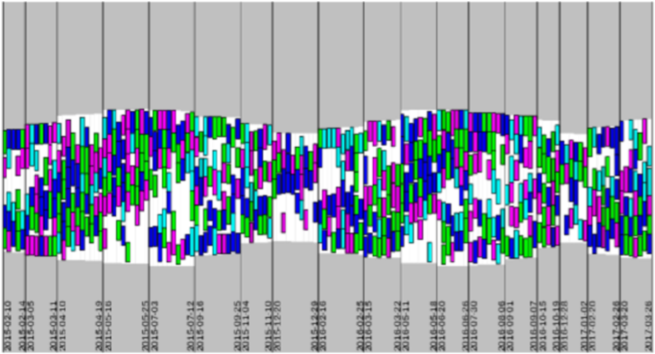
Nicolas Catusse, Hadrien Cambazard, Nadia Brauner, Pierre Lemaire, and Bernard Penz - Anne-Marie Lagrange and Pascal Rubini (IJCAI 2016) :

- Branch-and-price, local search

Model extensions:

- Observation duration depending on starting date
- Calibrations
- Night reservation
- **Variable interests**

The star scheduling problem

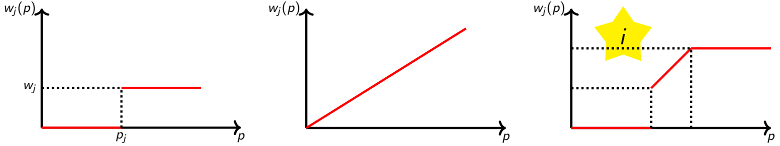


Processing-time dependent profit

Processing-time dependent profit

Instance: A set $\mathcal{N}$ of jobs. Each job $j \in \mathcal{N}$ has deadline d_j and a profit function $w_j(p_j)$, p_j the allocated processing-time of job j

Profit function for a classical scheduling problem, for the linear model and for a star observation:

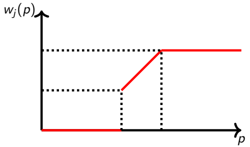


Question: find $\mathcal{N}' \subset \mathcal{N}$ such that $\sum_{j \in \mathcal{N}'} w_j(p_j)$ is maximized.

Processing-time dependent profit

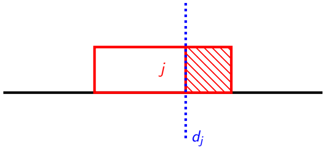
Controllable processing-times, Imprecise computation:

- Dvir Shabtay and George Steiner. A survey of scheduling with controllable processing times, 2007.



Processing-time dependent profit

- *Late work*:
  - Malgorzata Sterna. A survey of scheduling problems with late work criteria, 2011.



- *Increasing Reward with Increasing Service (IRIS)*:
  - Jayanta K Dey, James Kurose, and Don Towsley. On-line processor scheduling for class of iris (increasing reward with increasing service) real-time tasks, 1993.

Processing-time dependent profit

Goal: finding the NP-hardness limits for each model of the profit function depending on the machine environment and the constraints.

Machine environment: 1, Pm, P.

Constraints

- Release dates
- Preemption
- Identical parameters

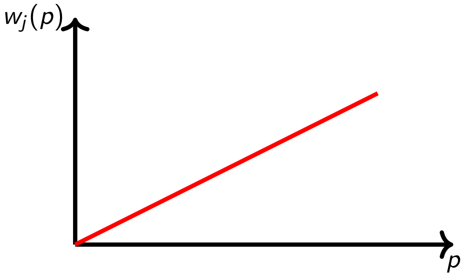
Simple examples

Simple examples

For all $j \in \mathcal{T}$:

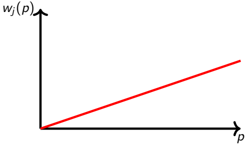
$$w_j(p) = b_j p$$

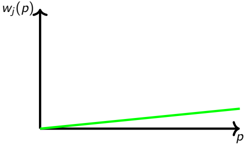
where b_j is called the growth rate of job j.

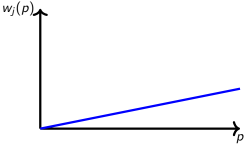


Simple examples

- No constraints, Common deadline:
  $1|w_j(p) = b_j p, d_j{=}d| - \sum w_j(p_j)$:







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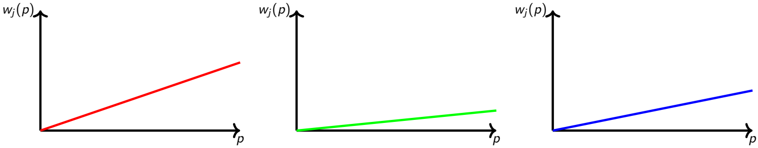

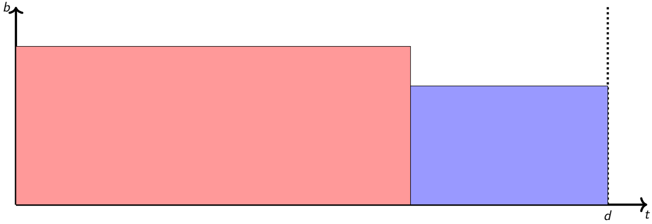

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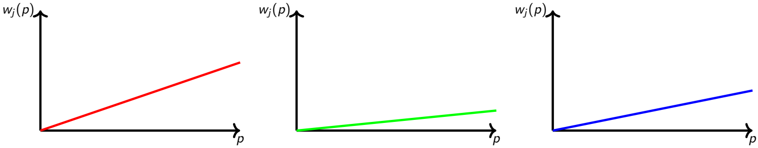

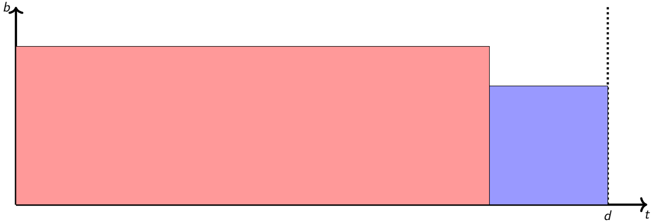

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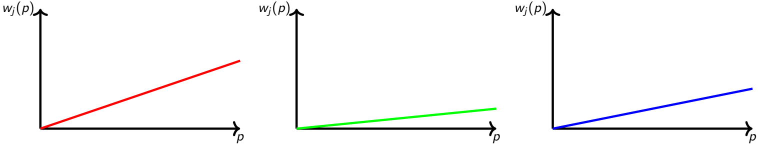
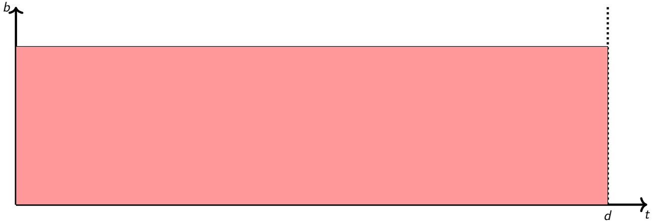
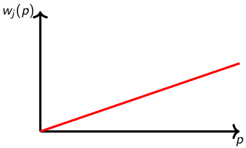


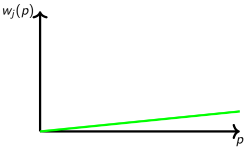
$\Longrightarrow$ Schedule only the job with the maximum growth rate b_j.

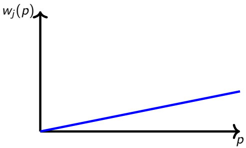


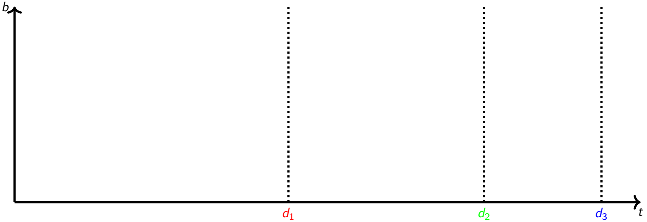
Simple examples

- Distinct deadlines $1|w_j(p) = b_j p| - \sum w_j(p_j)$:









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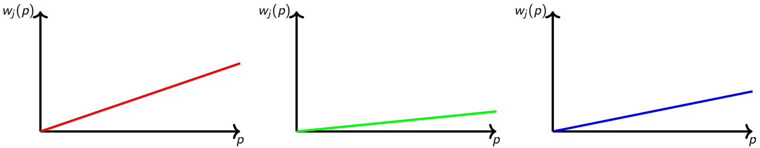

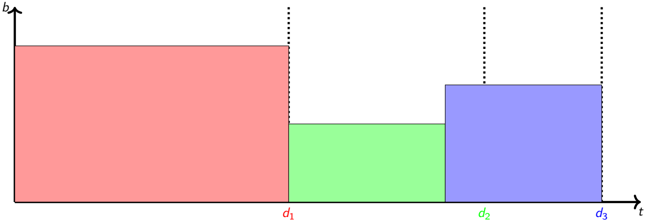

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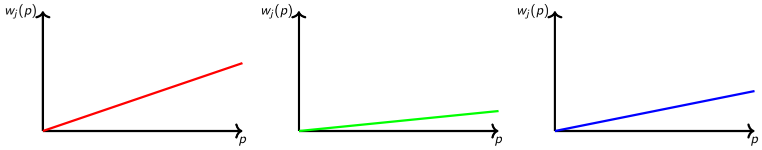
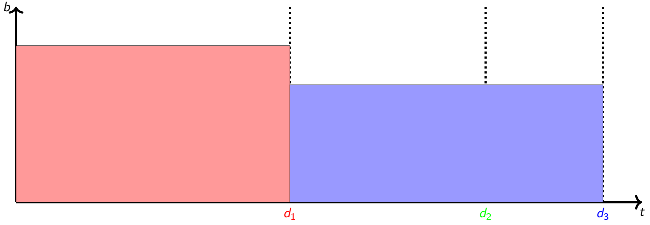


$\Longrightarrow$ Schedule the best available job till its deadline.



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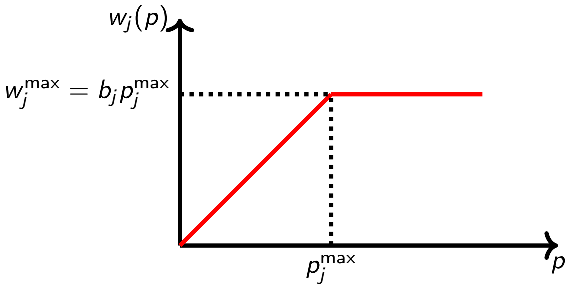
Simple examples

For all $j \in \mathcal{T}$:

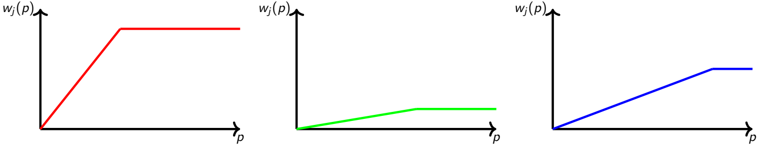
$$w_j(p) = \begin{cases} b_j p, & p \leq p_j^{\max} \\ w_j^{\max}, & p > p_j^{\max} \end{cases}$$

where $w_j^{\max}$ is called the maximum profit, b_j is called the growth rate and $p_j^{\max}$ the maximum processing-time of job j.



Simple examples

- No constraint, Common deadline
  $1|w_j(p) = \min\{b_j p, w_j^{\max}\}, d_j{=}d| - \sum w_j(p_j)$



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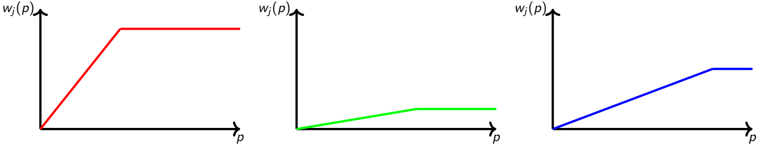
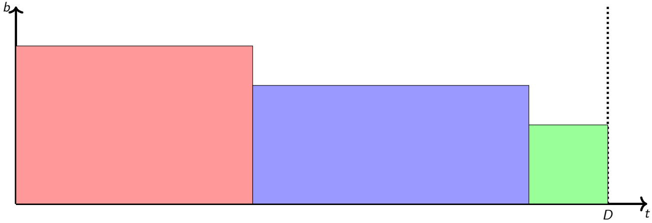


$\Longrightarrow$ Schedule jobs in non-increasing order of their growth rate



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Simple examples

- Distinct deadlines:
  $1|w_j(p) = \min\left\{b_j p, w_j^{\max}\right\}| - \sum w_j(p_j)$

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  $1|w_j(p) = \min\{b_j p, w_j^{\max}\} | -\sum w_j(p_j)$
  - The set of solutions for which every job is scheduled and in non-decreasing order of their deadline is dominant.

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Simple examples

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  - The set of solutions for which every job is scheduled and in non-decreasing order of their deadline is dominant.

$$\max \sum_{j=1}^{n} b_j p_j$$

$$\begin{aligned} s_j + p_j &\leq d_j & \forall j &= 1, \dots, n \\ s_{j+1} &= s_j + p_j & \forall j &= 1, \dots, n-1 \\ b_j p_j &\leq w_j^{\max} & \forall j &= 1, \dots, n \end{aligned}$$

Simple examples

- Release dates:
  $1|w_j(p) = \min\left\{b_j p, w_j^{\max}\right\}, r_j| - \sum w_j(p_j)$

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Strongly NP-complete, reduction from 3-Partition.

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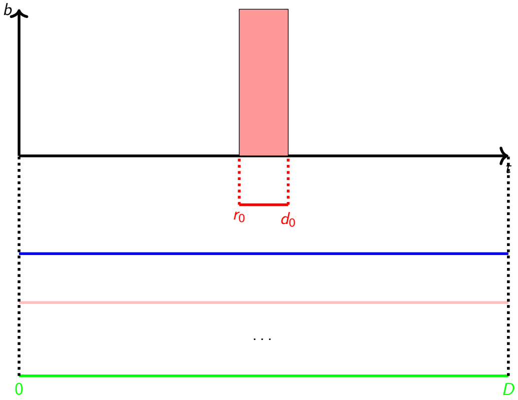

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Simple examples

- Release dates and preemption
  $1|w_j(p) = \min\left\{b_j p, w_j^{\max}\right\}, r_j, \text{pmtn}| - \sum w_j(p_j)$

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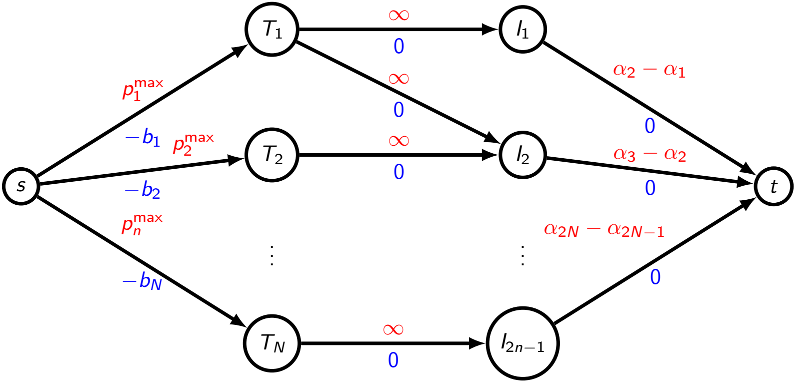

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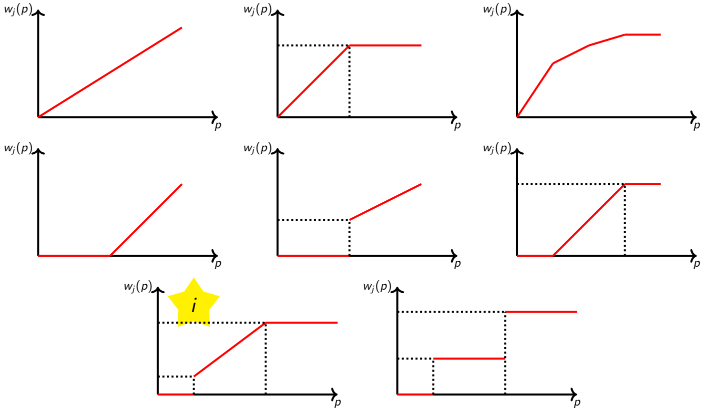


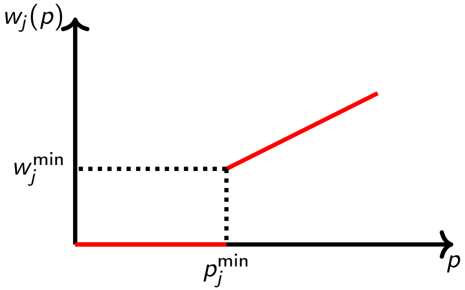
$P \mid \mathrm{LPSTIP}, p_j^{\mathsf{min}}{=}p^{\mathsf{min}}, d_j{=}d$

$P \mid \mathrm{LPSTIP}, p_j^{\min} = p^{\min}, d_j = d$

For all $j \in \mathcal{T}$:

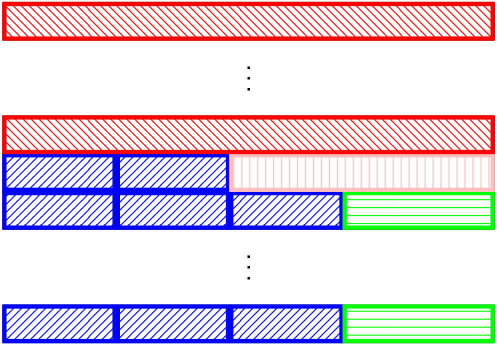
$$w_j(p) = \begin{cases} 0 & p < p_j^{\min} \\ w_j^{\min} + b_j(p - p_j^{\min}), & p \geq p_j^{\min} \end{cases}$$

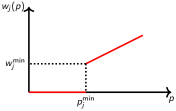
where $w_j^{\min}$ is called the minimum profit, b_j is called the growth rate and $p_j^{\min}$ the minimum processing-time of job j.



$P \mid \mathrm{LPSTIP}, p_j^{\min}{=}p^{\min}, d_j{=}d$

Dominant structure of solutions:





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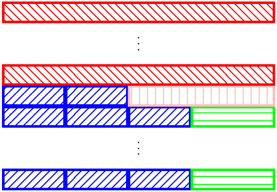

$$
\begin{aligned}
U_1(S) &= \{j \in S, \quad p_j = D\} \\
U_2(S) &= \{j \in S, \quad T_j \text{ is a special job}\} \\
U_3(S) &= \{j \in S, \quad p_j = p^{\min} + r\} \\
U_4(S) &= \{j \in S, \quad p_j = p^{\min}\} \\
U_0(S) &= \{j \notin S\}
\end{aligned}
$$

$$
t(S) = \begin{cases} 0, & \text{if } U_2(S) = \emptyset \\ \frac{D - p_j}{p^{\min}}, & \text{if } U_2(S) = \{T_j\} \end{cases}
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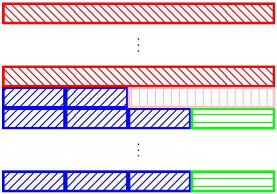

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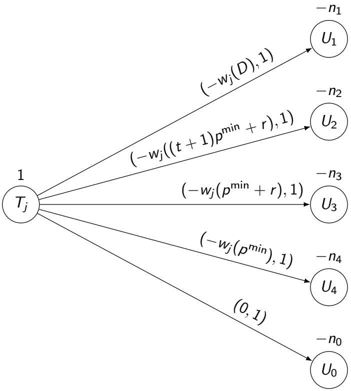

$P \mid \mathrm{LPSTIP}, p_j^{\mathbf{min}}{=}p^{\mathbf{min}}, d_j{=}d$

$$0 \leq N_1(S) \leq m \qquad 0 \leq t(S) \leq n-1$$

$P \mid \mathrm{LPSTIP}, p_j^{\min}=p^{\min}, d_j=d$

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$$N_2(S) = \begin{cases} 0, & \text{if } t(S) = 0 \\ 1, & \text{otherwise} \end{cases}$$

$$N_3(S) = m - N_1(S) - N_2(S)$$

$$N_4(S) = (q-1)N_3(S) + t(S)$$

$$N_0(S) = N - N_1(S) - N_2(S) - N_3(S) - N_4(S)$$

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$\Longrightarrow$ The problem can be solved in polynomial time by solving a polynomial number of minimum cost flows

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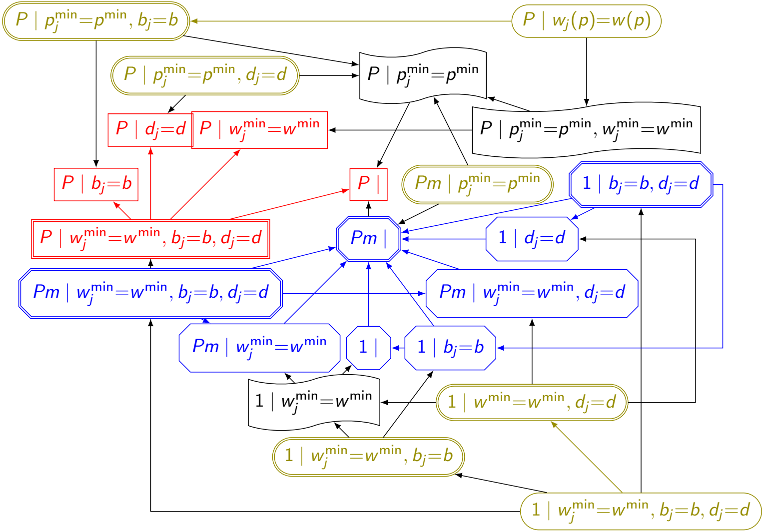
$P \mid \text{LPSTIP}, p_j^{\min}=p^{\min}, d_j=d$



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$P \mid \mathrm{LPSTIP}, p_j^{\min}{=}p^{\min}, d_j{=}d$

Problem	Compl.	Ref
$P \mid \mathrm{LP}$	P	Th. 1
$P \mid \mathrm{LBP}, b_j{=}b, d_j{=}d$	sNPc	Th. 3
$Pm \mid \mathrm{LBP}, b_j{=}b, d_j{=}d$	wNPc	Th. 3
$P \mid \mathrm{LBP}, w_j^{\max}{=}w^{\max}, d_j{=}d$	sNPc	Th. 3
$Pm \mid \mathrm{LBP}, w_j^{\max}{=}w^{\max}, d_j{=}d$	wNPc	Th. 3
$1 \mid \mathrm{LBP}$	P	Th. 2
$P \mid \mathrm{LBP}, w_j(p){=}w(p), d_j{=}d$	P	Th. 4
$P \mid \mathrm{LPST}, b_j{=}b$	P	Th. 5
$P \mid \mathrm{LPSTIP}, w_j^{\min}{=}w^{\min}, b_j{=}b, d_j{=}d$	sNPc	Th. 6
$Pm \mid \mathrm{LPSTIP}, w_j^{\min}{=}w^{\min}, b_j{=}b, d_j{=}d$	wNPc	Th. 6
$1 \mid \mathrm{LPSTIP}, b_j{=}b, d_j{=}d$	wNPc	Th. 6
$P \mid \mathrm{LPSTIP}, p_j^{\min}{=}p^{\min}, d_j{=}d$	P	Th. 7
$P \mid \mathrm{LPSTIP}, p_j^{\min}{=}p^{\min}, b_j{=}b$	P	Th. 8
$P \mid \mathrm{LBPST}, w_j^{\max}{=}w^{\max}, p_j^{\min}{=}p^{\min}, d_j{=}d$	sNPc	Th. 9
$Pm \mid \mathrm{LBPST}, w_j^{\max}{=}w^{\max}, p_j^{\min}{=}p^{\min}, d_j{=}d$	wNPc	Th. 9
$P \mid \mathrm{LBPST}, p_j^{\max}{=}p^{\max}, d_j{=}d$	P	Th. 11
$1 \mid \mathrm{LBPST}, b_j{=}b, d_j{=}d$	wNPc	Th. 10
$1 \mid \mathrm{LBPST}, b_j{=}b, p_j^{\min}{=}p^{\min}, d_j{=}d$	P	
$P \mid \mathrm{LBPSTIP}, w_j^{\min}{=}w^{\min}, b_j{=}b, p_j^{\max}{=}p^{\max}, w_j^{\max}{=}w^{\max}, d_j{=}d$	sNPc	Th. 12
$Pm \mid \mathrm{LBPSTIP}, w_j^{\min}{=}w^{\min}, b_j{=}b, p_j^{\max}{=}p^{\max}, w_j^{\max}{=}w^{\max}, d_j{=}d$	wNPc	Th. 12
$1 \mid \mathrm{LBPSTIP}, b_j{=}b, w_j^{\max}{=}w^{\max}, d_j{=}d$	wNPc	Th. 13
$1 \mid \mathrm{LBPSTIP}, p_j^{\max}{=}p^{\max}, w_j^{\max}{=}w^{\max}, d_j{=}d$	wNPc	Th. 13
$Pm \mid \mathrm{PLP}, p_j^k{=}p^k$	P	Th. 14
$1 \mid \mathrm{PLP}, w_j^k{=}w^k, b_j^k{=}b$	P	Th. 15
$1 \mid \mathrm{PLP}, w_j^k{=}w^k, b_j^k{=}b_j, d_j{=}d$	P	Th. 16

G-SCOP

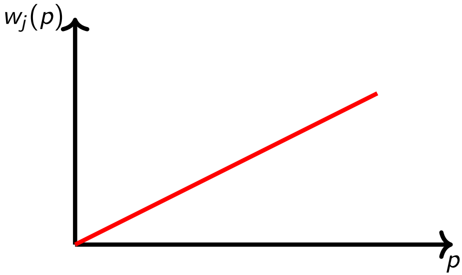
Linear model with release dates

Linear model with release dates

For all $j \in \mathcal{T}$:

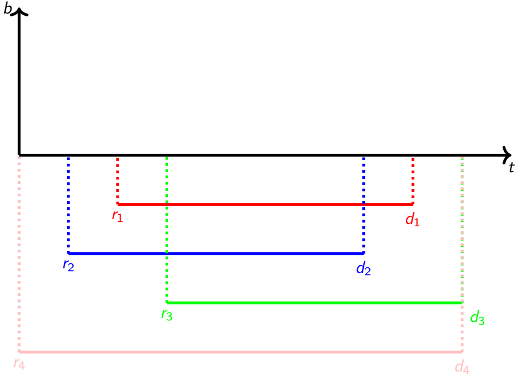
$$w_j(p) = b_j p$$

where b_j is called the growth rate of job j.



Linear model with release dates

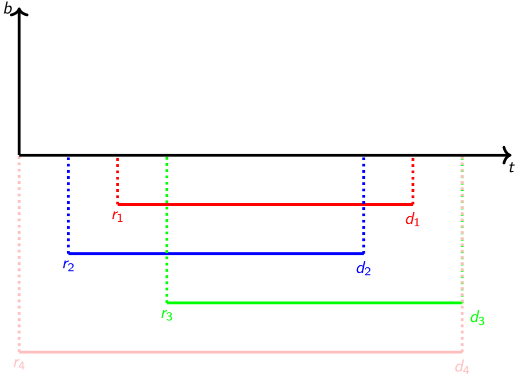
Dominance: every scheduled job starts and ends on a r_j or a d_j (but not necessarily its own).



Linear model with release dates

Dominance: every scheduled job starts and ends on a r_j or a d_j (but not necessarily its own).

Schedule jobs in non-increasing order of their growth rates?



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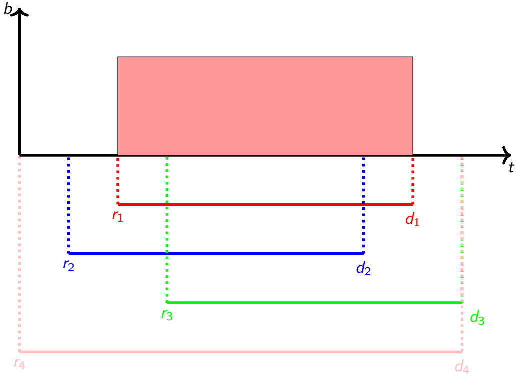

G-SCOP

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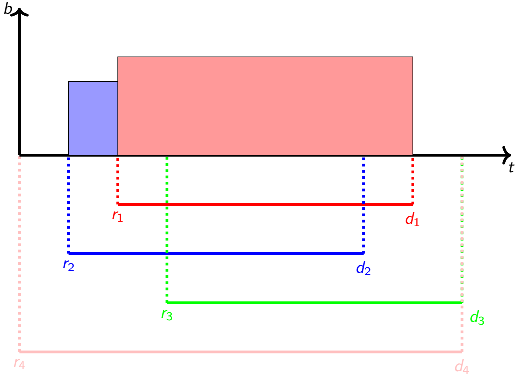

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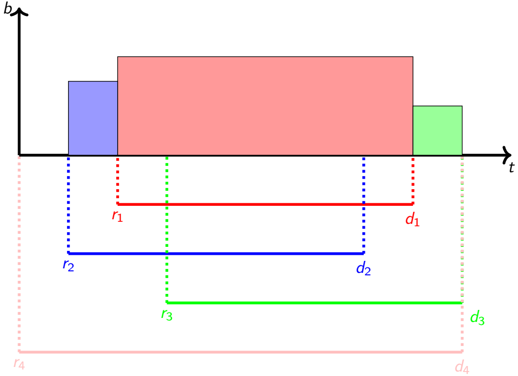

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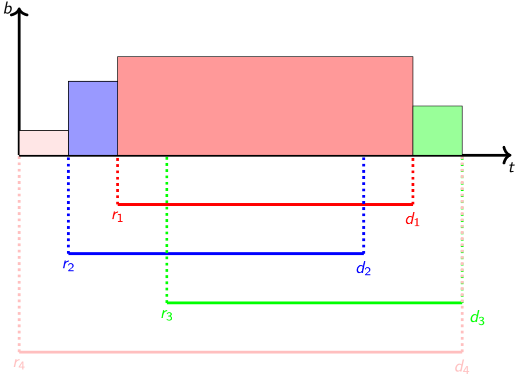

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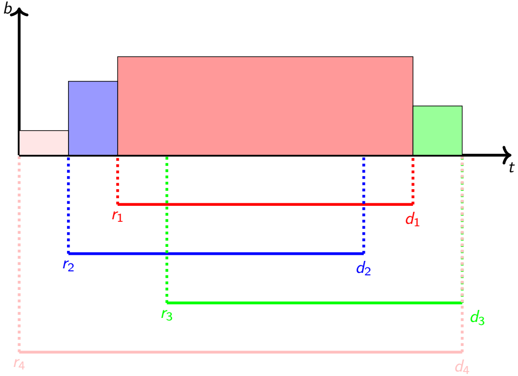

$\Longrightarrow$ Optimal solution for this instance

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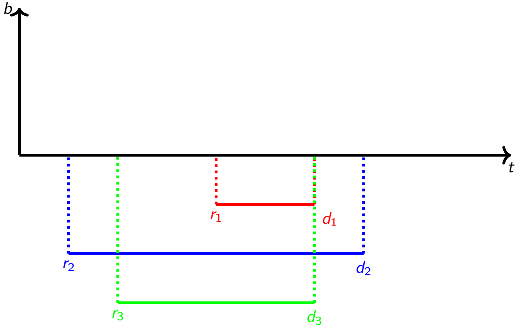
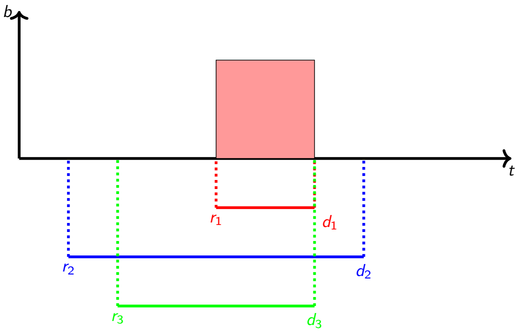
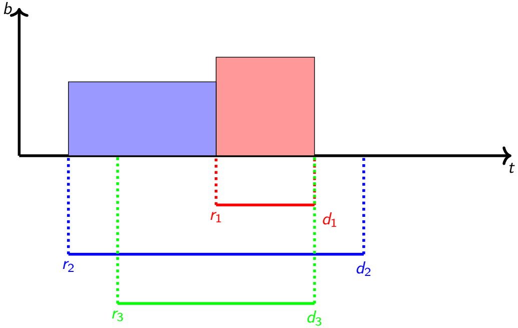
Linear model with release dates



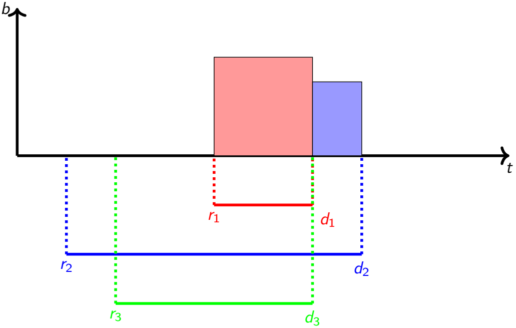
Linear model with release dates



Linear model with release dates



Linear model with release dates



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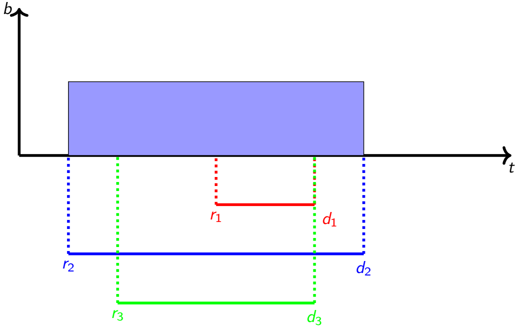
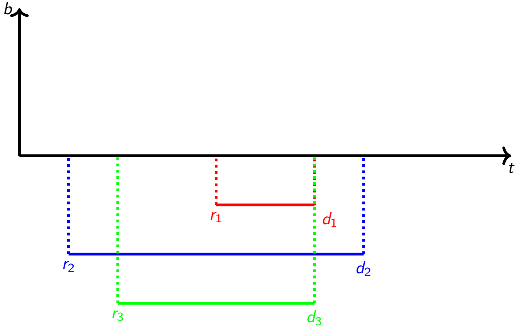
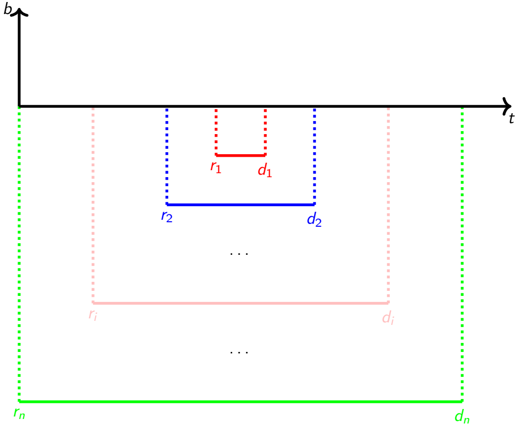


Linear model with release dates

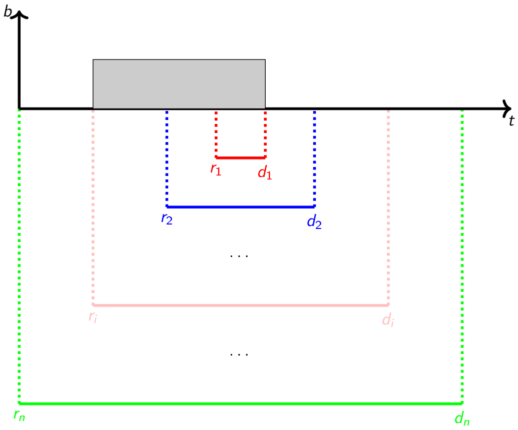


$\Longrightarrow$ a job which is the best on its time-window is either scheduled on its whole time-window, or not scheduled at all.

Linear model with release dates



Linear model with release dates



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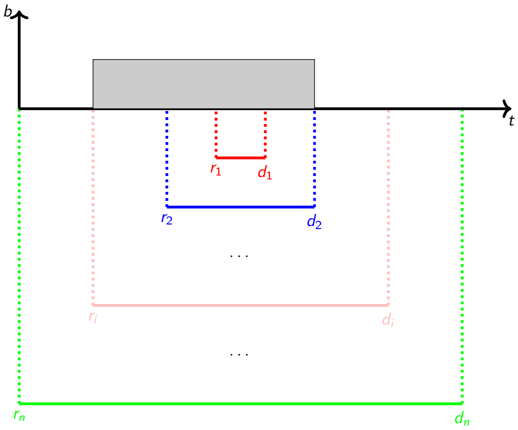

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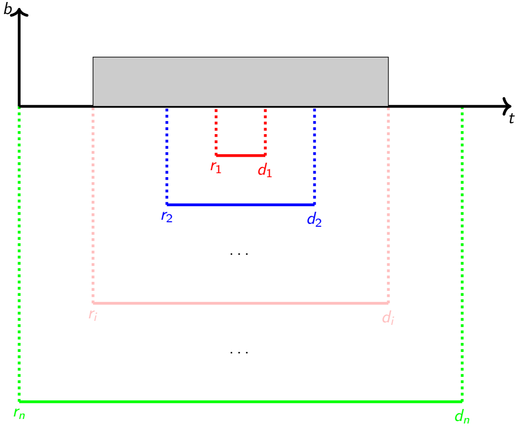

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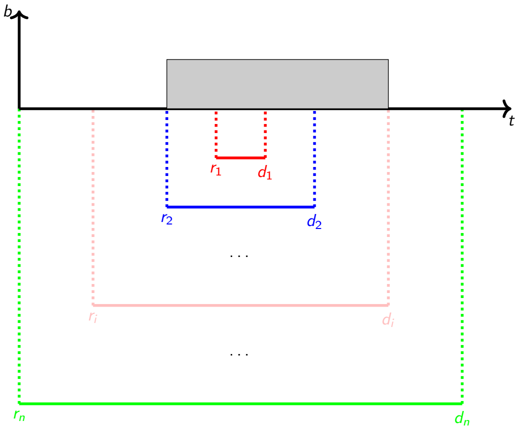

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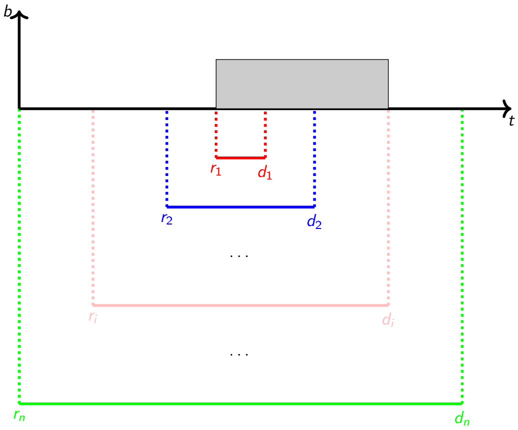

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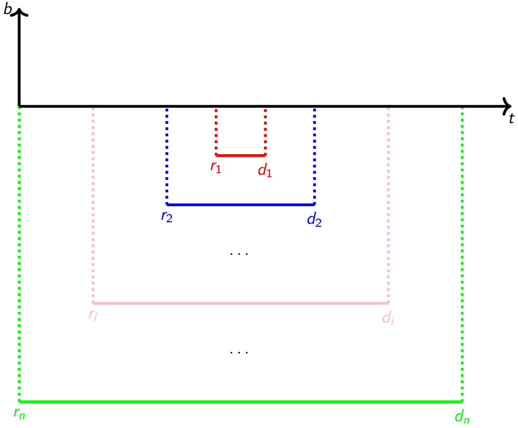
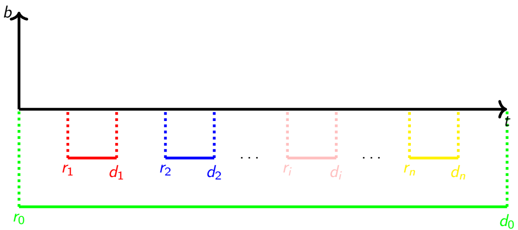


At step j, $O(j^2)$ possible states
$\Longrightarrow$ solved in polynomial time by a shortest path algorithm

Linear model with release dates



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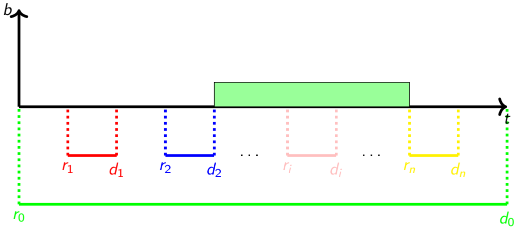
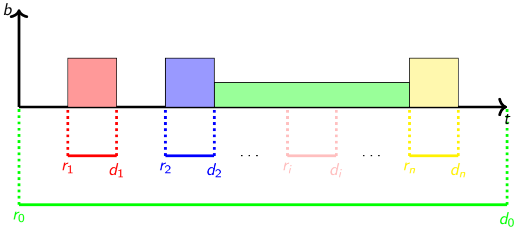


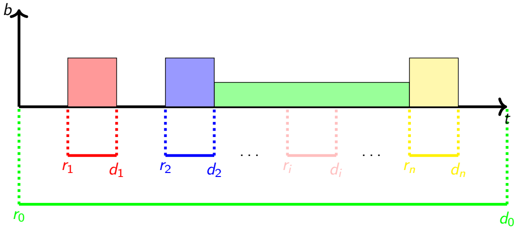
$O(n^2)$ possible schedules for job T_0.

Linear model with release dates



$O(n^2)$ possible schedules for job T_0.
Once the schedule of T_0 is fixed, the problem can be trivially solved in polynomial time.

Linear model with release dates



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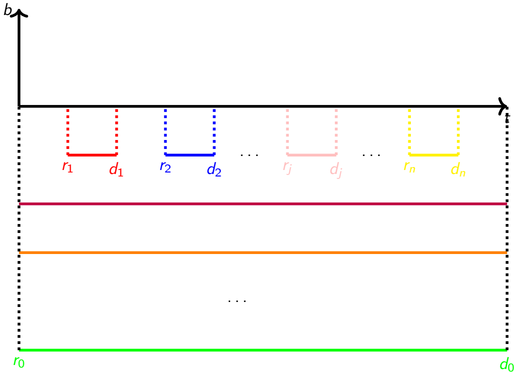
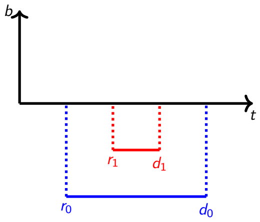
$\Longrightarrow$ solved in polynomial time

Linear model with release dates



Linear model with release dates

- A vertex for each possible schedule of each job with weight equal to the profit of the schedule
  $\implies O(n^3)$ vertices
- An edge between two vertices iff the corresponding schedules are not compatible (same job or common instant)



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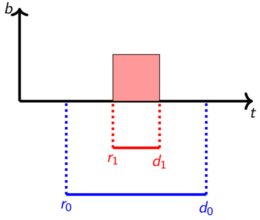

G-SCOP

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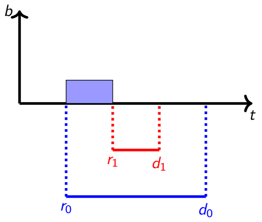

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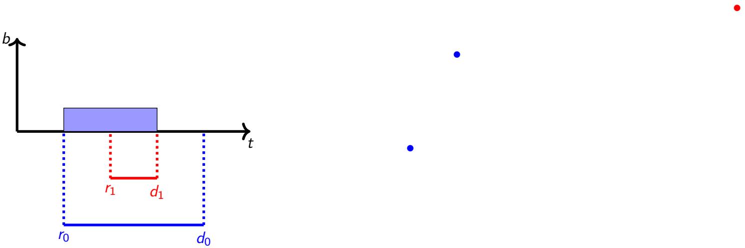

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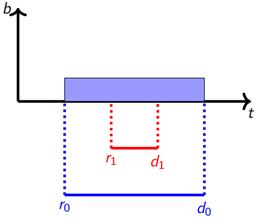

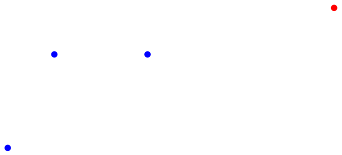

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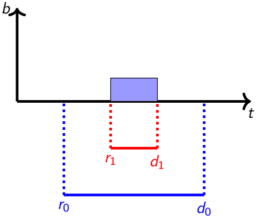

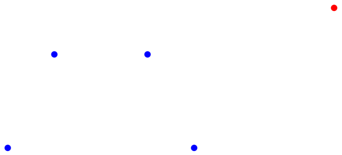

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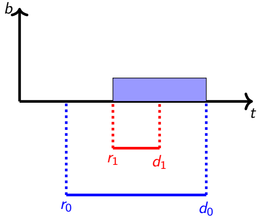

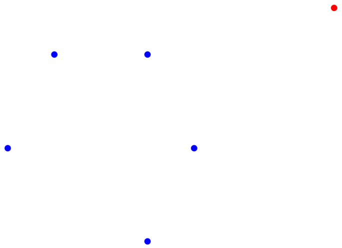

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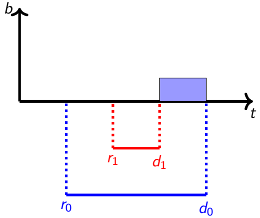

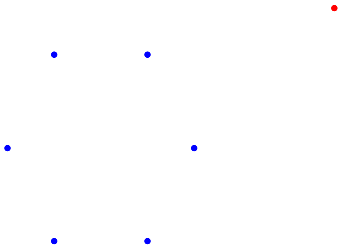

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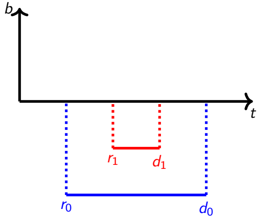

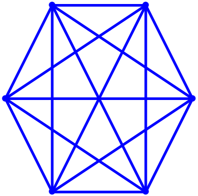

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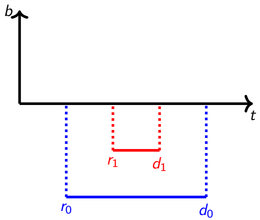
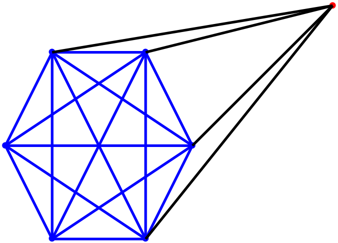




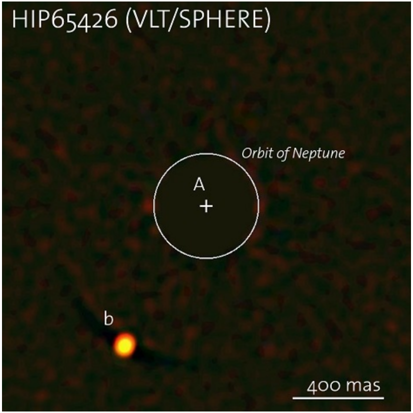
Linear model with release dates

G-SCOP

Conclusion

- Interesting class of scheduling problems
- Various resolution methods: list algorithms, linear programming, flows, classical reductions. . . (good exercices for students!)
- Some "basic" cases still open

- Integration in the practical problem
- Integration in other problem models to improve accuracy or speed up resolution.



Questions?