Merging and Memorization in search trees : on the exact solution of scheduling problems

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Outline

1. Problem: $1||\sum T_i$
2. Branch & Merge (theoretical guarantee)
3. Memorization (practical efficiency)
4. Extension : Branch & Memorize framework on sequencing problems

Problem: $1||\sum T_i$

- Jobset S, single machine, p_j=processing time, d_j=due date
- Objective: minimize the total tardiness $\sum_j \max(0, C_j - d_j)$
- NP-hard (ordinary sense)
- In theory (complexity):
  - Brute force $O(n!)$
  - Dynamic programming: $O^*(2^n)$ in time and space
  - Divide & Conquer: $O^*(4^n)$, polynomial space (Gurevich et al., 1987)
  - Branch & Reduce: $O^*(3^n)$, polynomial space (F. D. Croce et al, 2015)
  - Branch & Merge => $O^*((2+\epsilon)^n)$ in time and polynomial space

- In practice:
  - The B&B of Szwarc et al. => 500 jobs in 2001. (900 jobs today!)
  - Memorization => 1200 jobs.

In Theory

Objective

- Exact algorithms with worst-case running time/space guarantee ($O^*(c^n)$, with c a constant as small as possible)

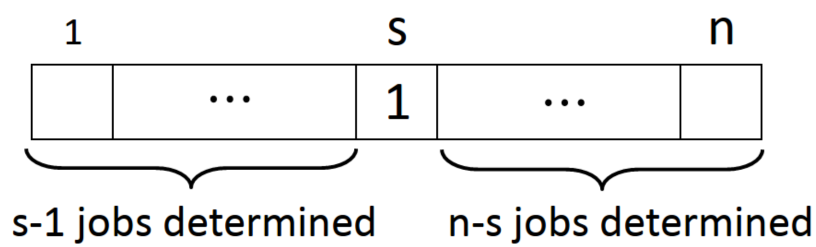
Notation

- LPT (Longest Processing Time first) job sequence: $(1,2,..,n)$
- EDD (Earliest Due Date first) job sequence: $(e_1, e_2, .., e_n)$

In Theory

Lawler's Property (1977)

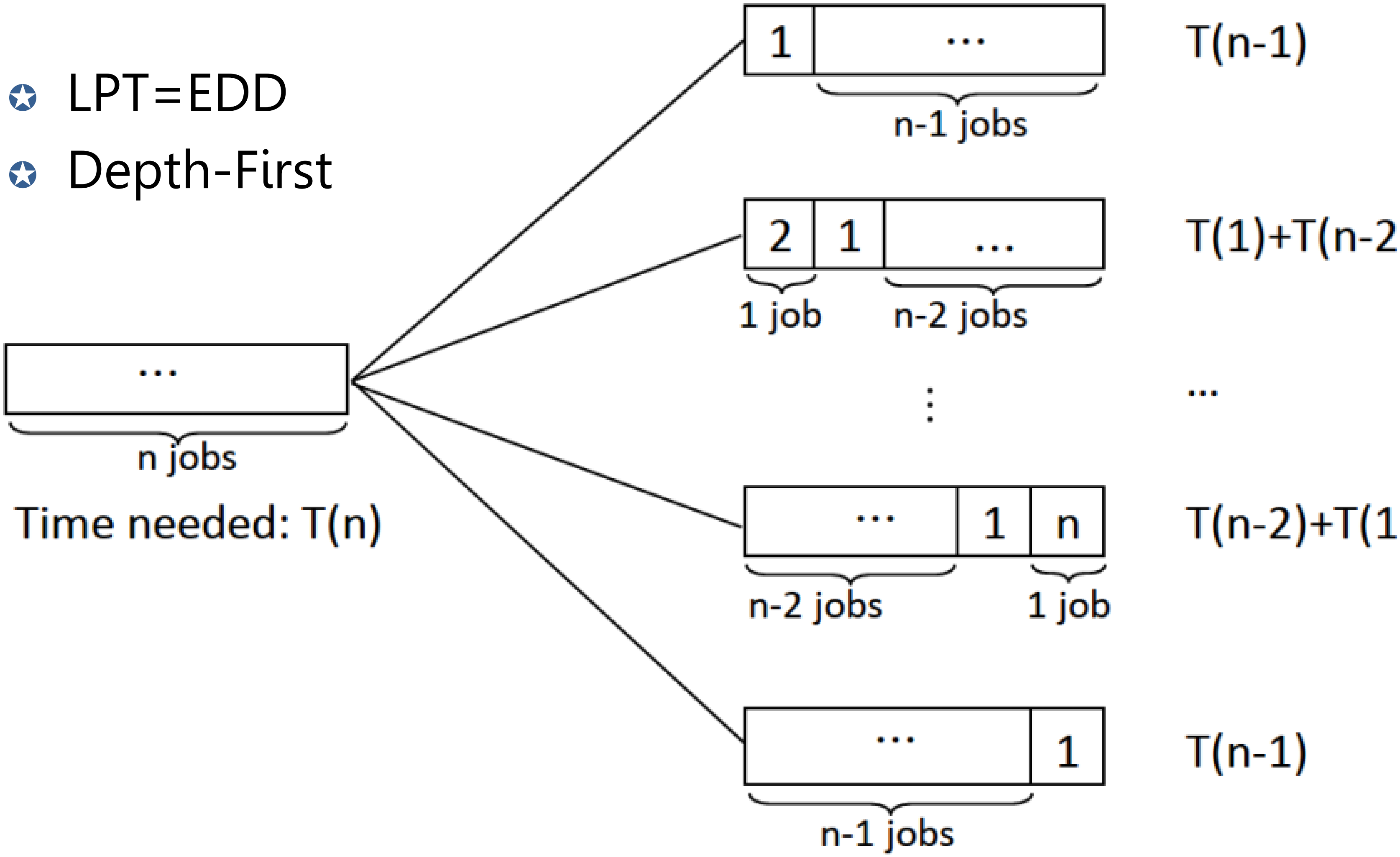
- Let job 1 $= e_h$, then job 1 can only be set in position $s \geq h$
- Jobs preceding 1 are: $B_1 = \{e_1, e_2, .., e_{h-1}, e_{h+1}, .., e_s\}$
- Jobs following 1 are: $A_1 = \{e_{s+1}, e_{s+2}, .., e_n\}$



- => Worst case: LPT=EDD

Branch & Reduce

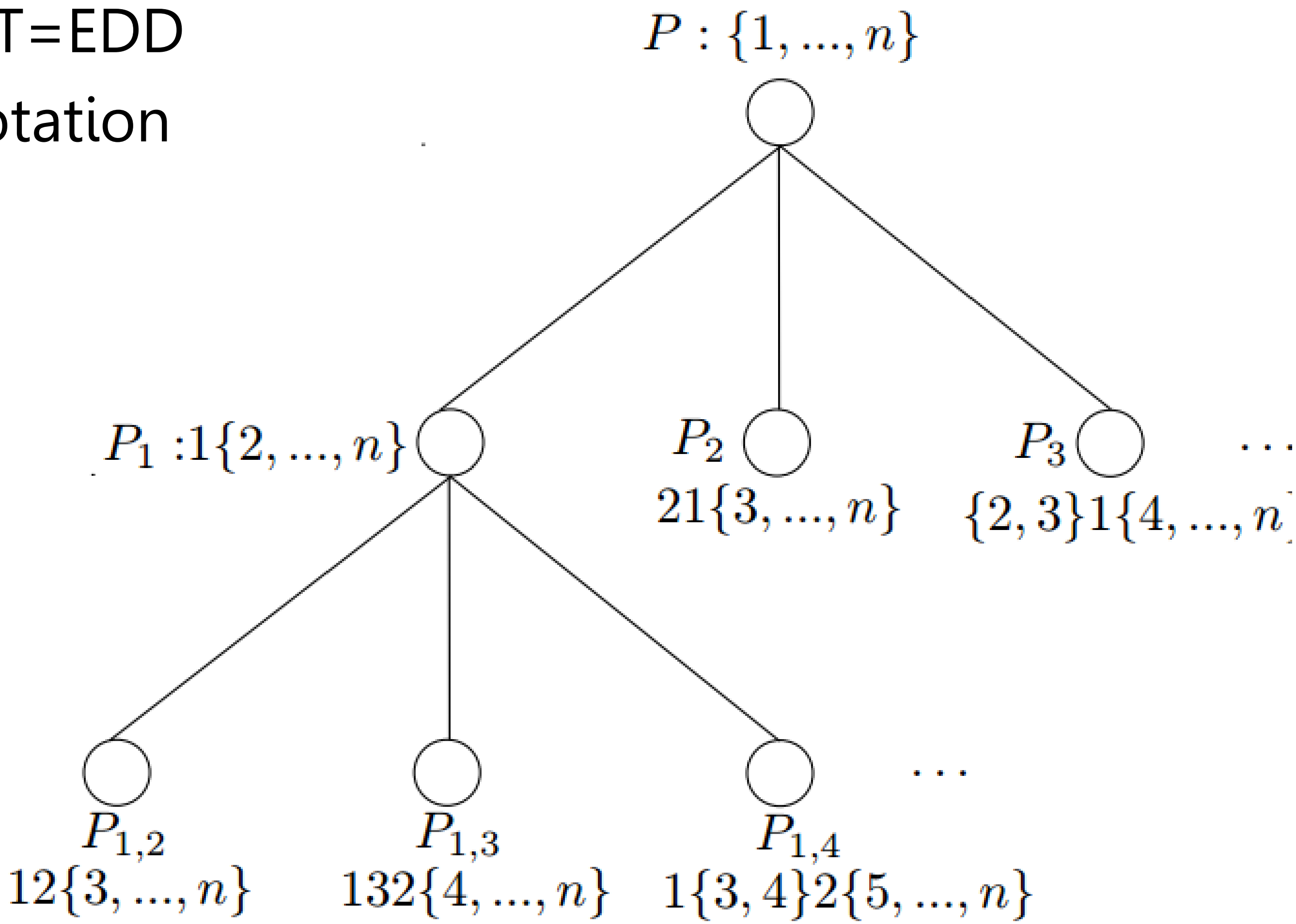
- LPT=EDD
- Depth-First



$$T(n) \leq 2\,T(n\text{-}1) + 2T(n\text{-}2) + \cdots + 2\,T(1) \Rightarrow T(n) = O(3^n)$$

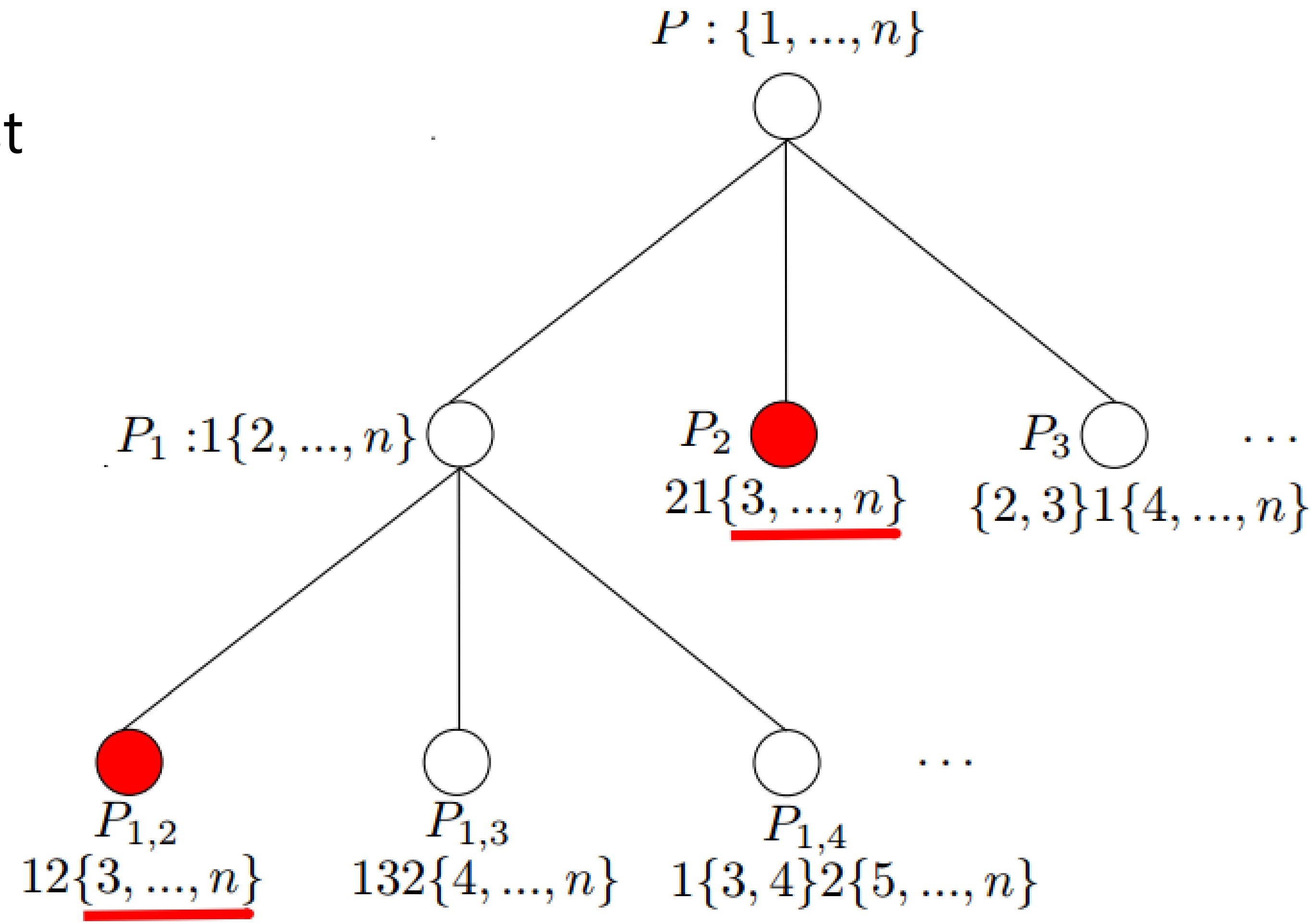
Branch & Reduce

- LPT=EDD
- Notation



Branch & Reduce: observations

- LPT=EDD
- Depth-First



- **Some sub-problems are solved repeatedly!**

Branch & Reduce: observations

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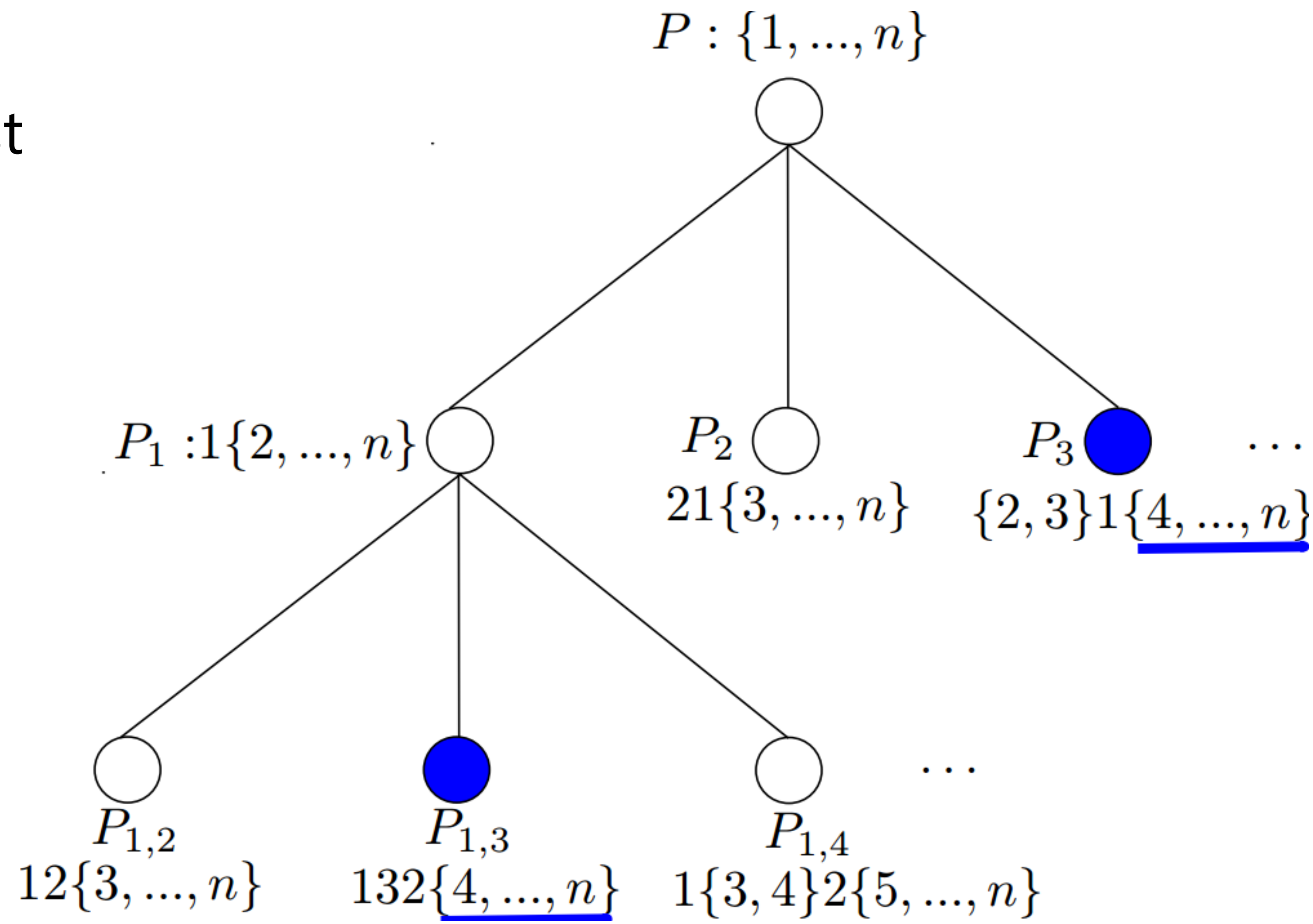
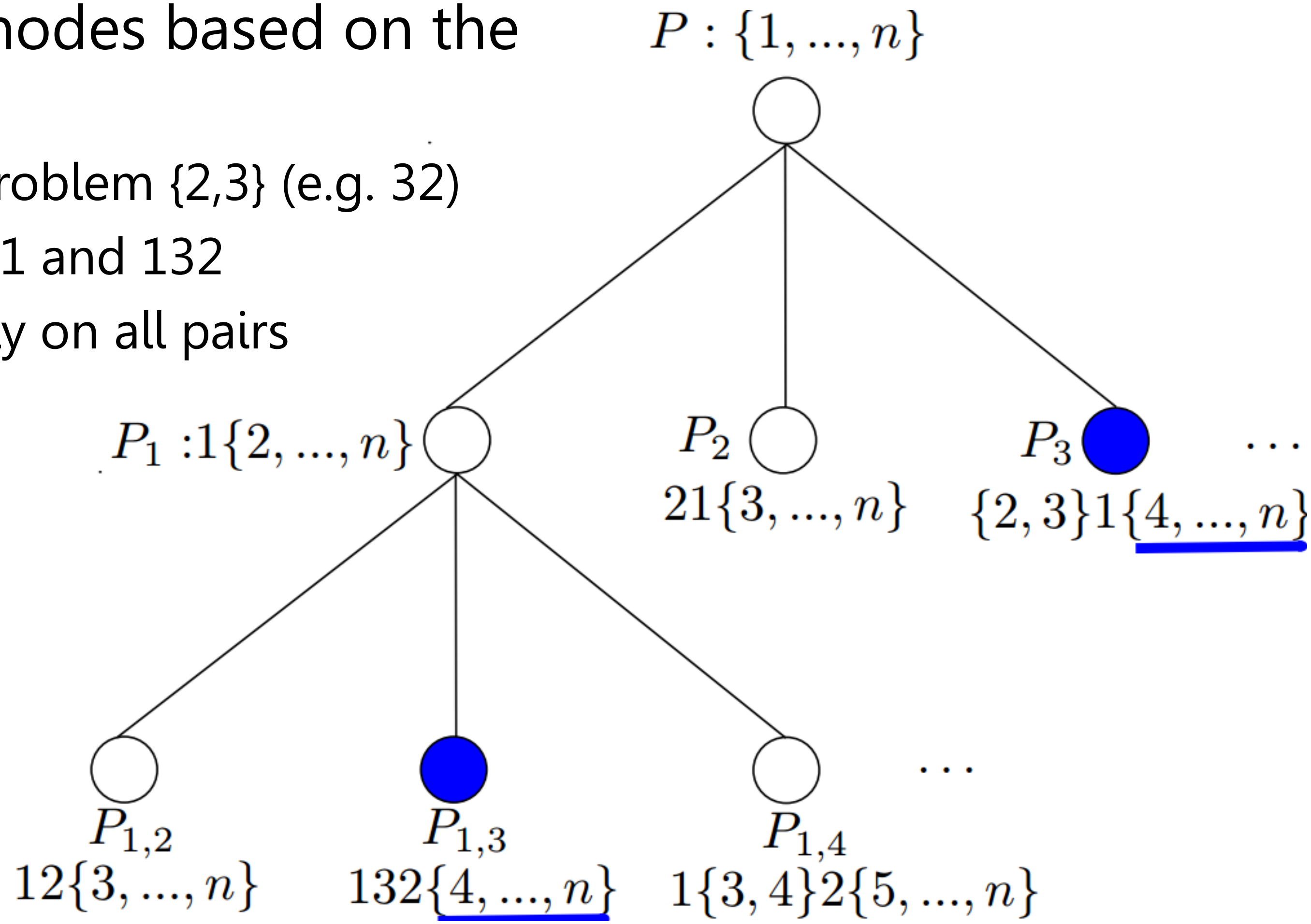
- Depth-First



- **Some sub-problems are solved repeatedly!**

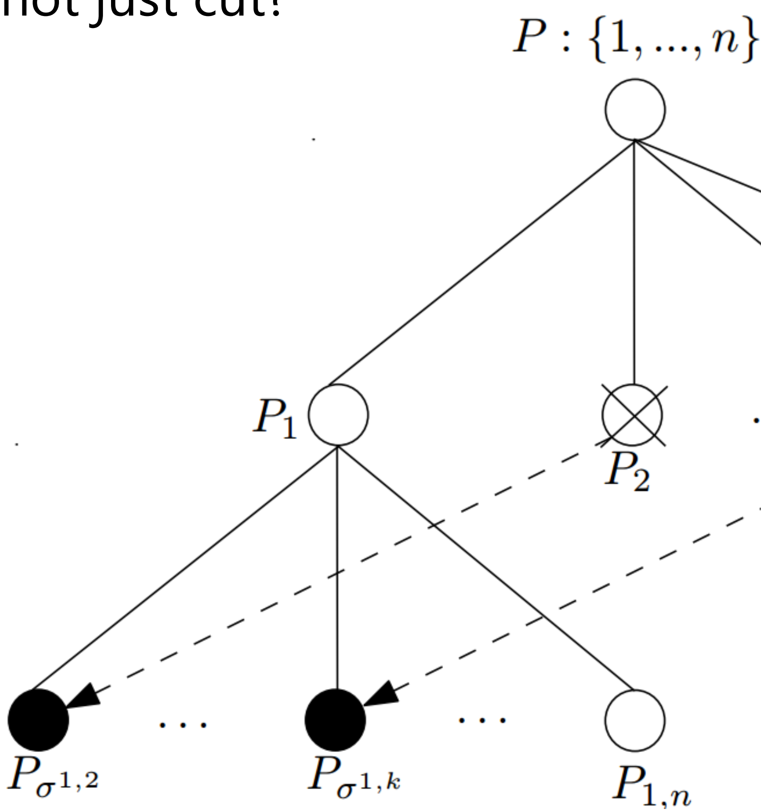
Branch & Merge (left)

- Idea: merge nodes based on the fixed part
  - Solve sub-problem {2,3} (e.g. 32)
  - Compare 321 and 132
  - Cannot apply on all pairs



Branch & Merge (left)

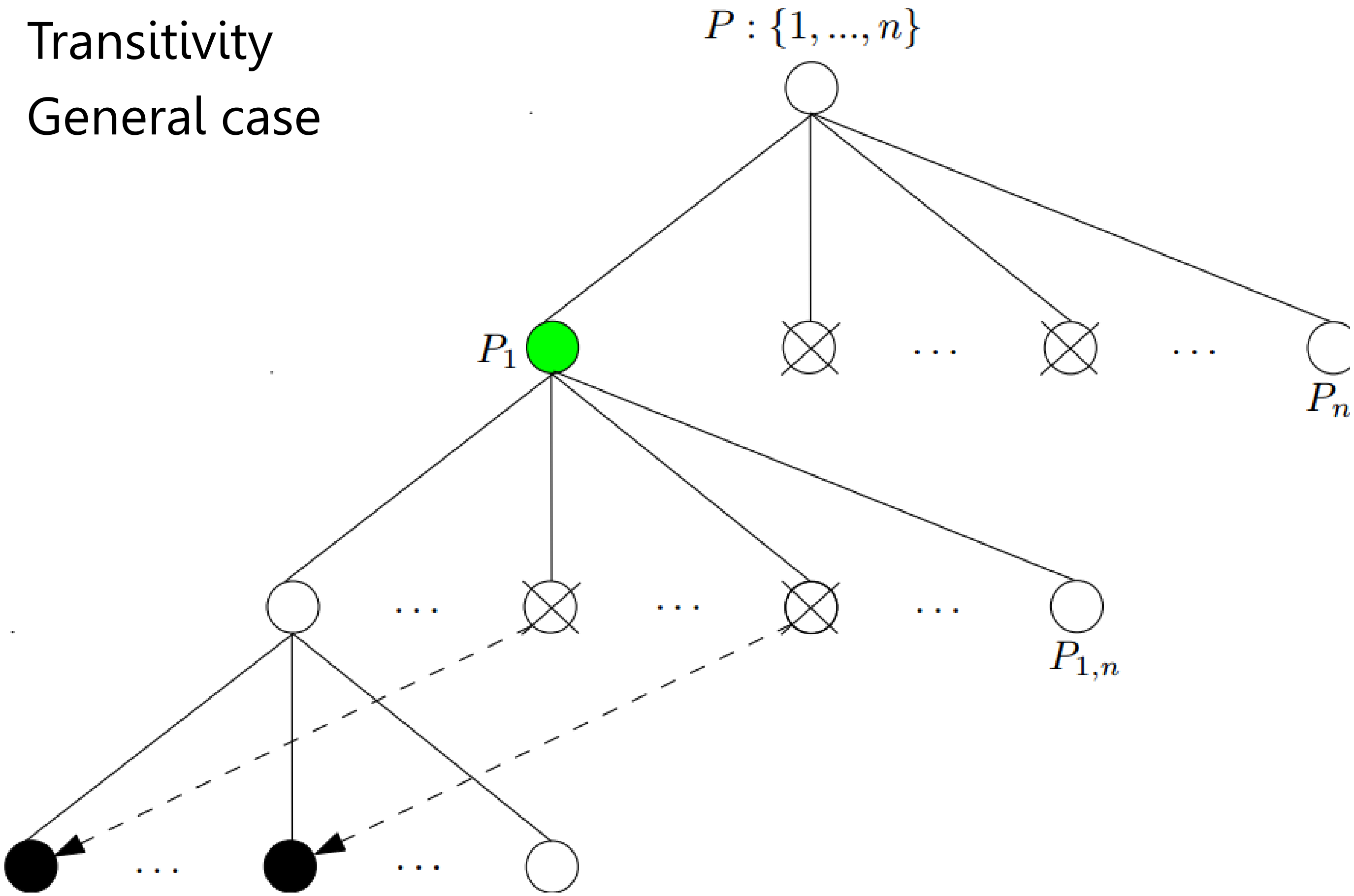
- Idea: merge identical nodes based on the fixed part
  - On first k nodes, k is a constant
  - Why not just cut?



P_1 :$1\{2, ..., n\}$
P_2 :$21\{3, ..., n\}$
P_k :$\{2..k\}1\{k+1..n\}$
$P_{\sigma 1,2}$:$\text{BEST}(12, 21)\{3..n\}$
$P_{\sigma 1,k}$:$\text{BEST}(\{2..k\}1, 1\{3..k\}2)\{k+1..n\}$

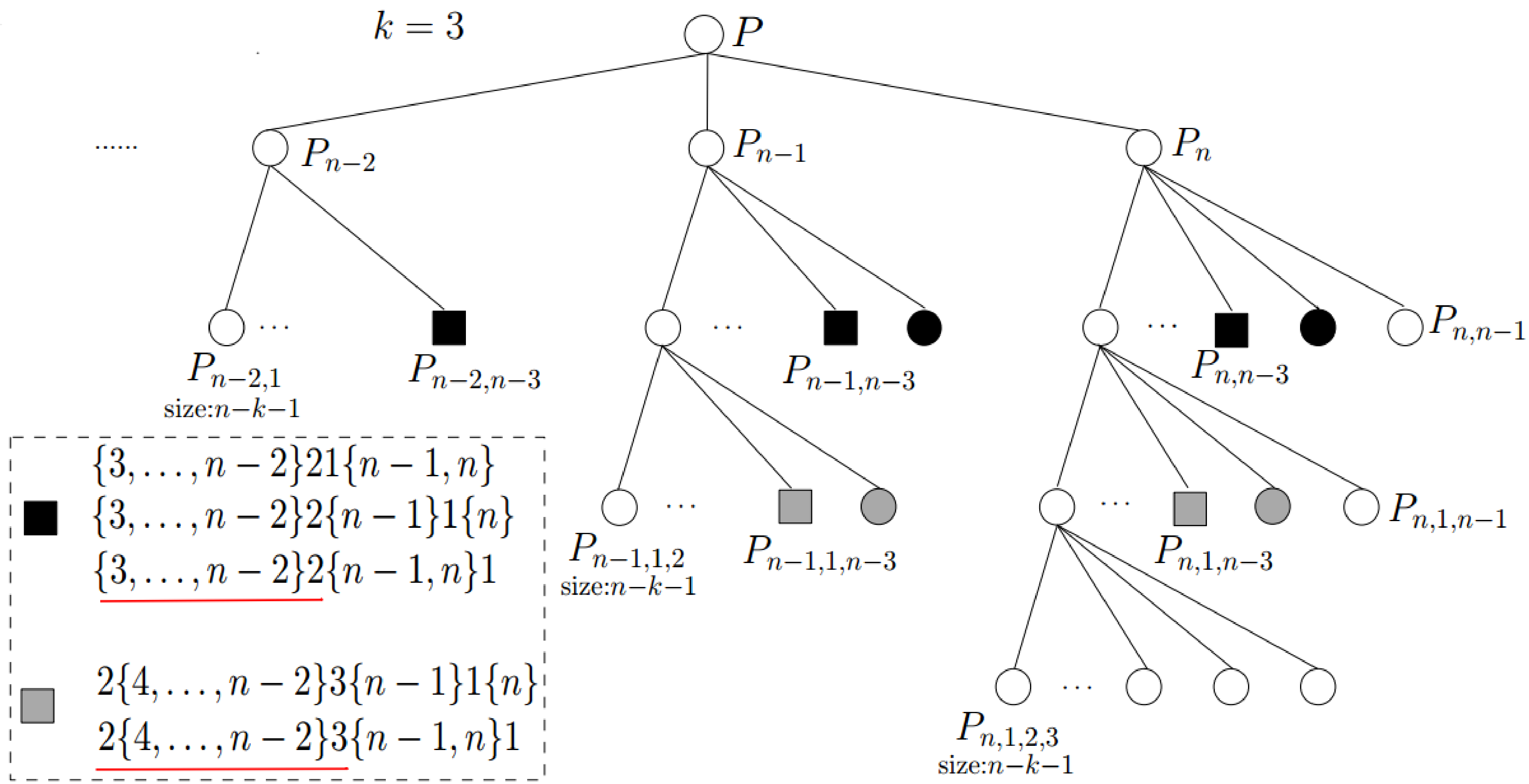
Branch & Merge (left)

- Idea: merge identical nodes based on the fixed part
  - On first k nodes, k is a constant
  - Transitivity
  - General case

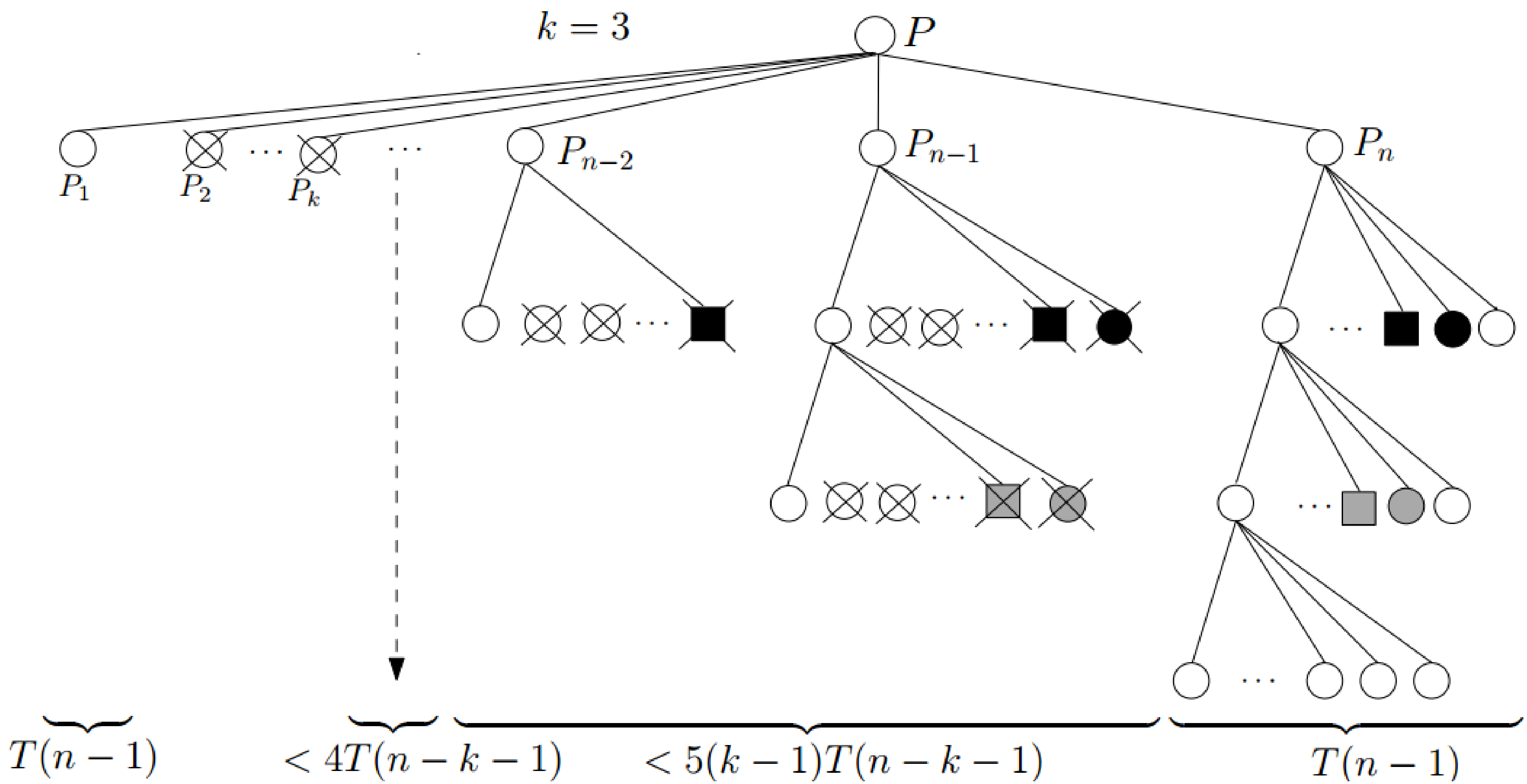


Branch & Merge (right)

- More complex…



Branch & Merge



- Recurrence: $T(n) \leq 2T(n-1) + (5k-1)T(n-k-1) + O(p(n))$

Branch & Merge

- $T(n)$ converges to $O^*(2^n)$. $T(n) = O^*(2.0367^n)$ when $k = 10$

k	$T(n)$
5	$\mathcal{O}^*(2.3065^n)$
10	$\mathcal{O}^*(2.0367^n)$
15	$\mathcal{O}^*(2.0022^n)$
20	$\mathcal{O}^*(2.0001^n)$

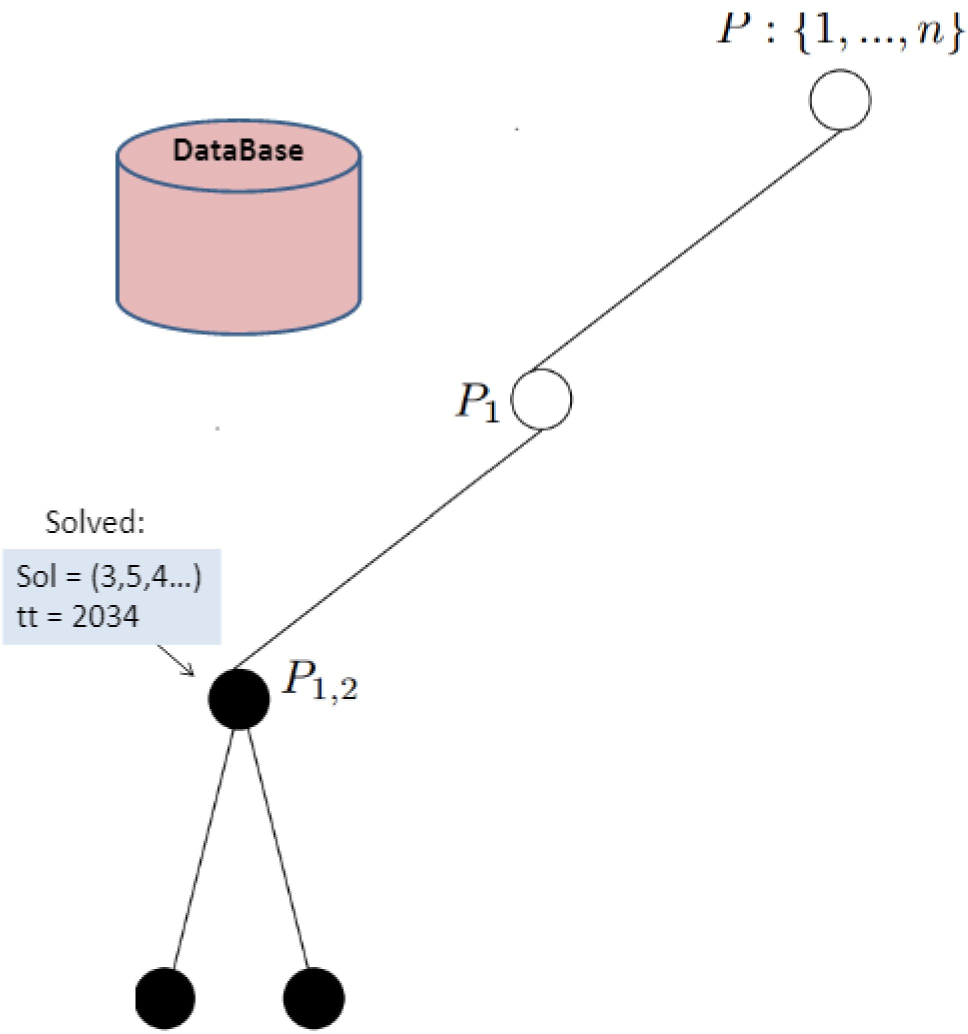
Summary 1

- Branch & Merge in ~ $O^*(2^n)$ time and polynomial space
- Can be generalized to other problems: branch smartly
- Work done together with:
  - Federico Della Croce
  - Vincent T'Kindt
  - Michele Garraffa

In Practice

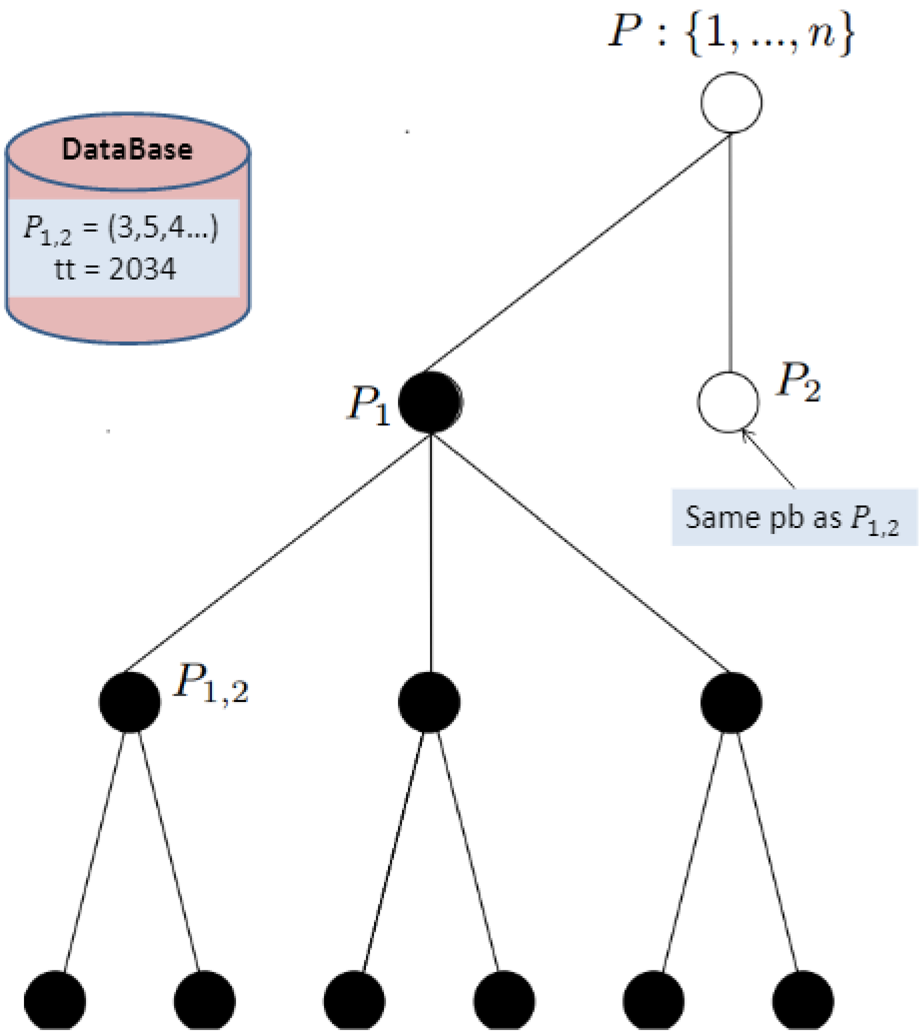
- BB2001: Szwarc et al. 2001
  - Solved 500 jobs in 2001 (900 jobs today!)
  - **Split**: decompose by precedence relations
  - **PosElim**: eliminates bad branching positions
  - **Memorization**: avoids solving a problem twice by storing its solution (basically merging without moving nodes)
- Without Split, PosElim
  - Branch & Merge is clearly more efficient than Branch & Reduce
- With Split, PosElim
  - Split & PosElim: break the structure of merging
- Memorization is more practical, even though theoretically exponential space.

In Practice: Memorization



- « Never solve a problem twice »

In Practice: Memorization



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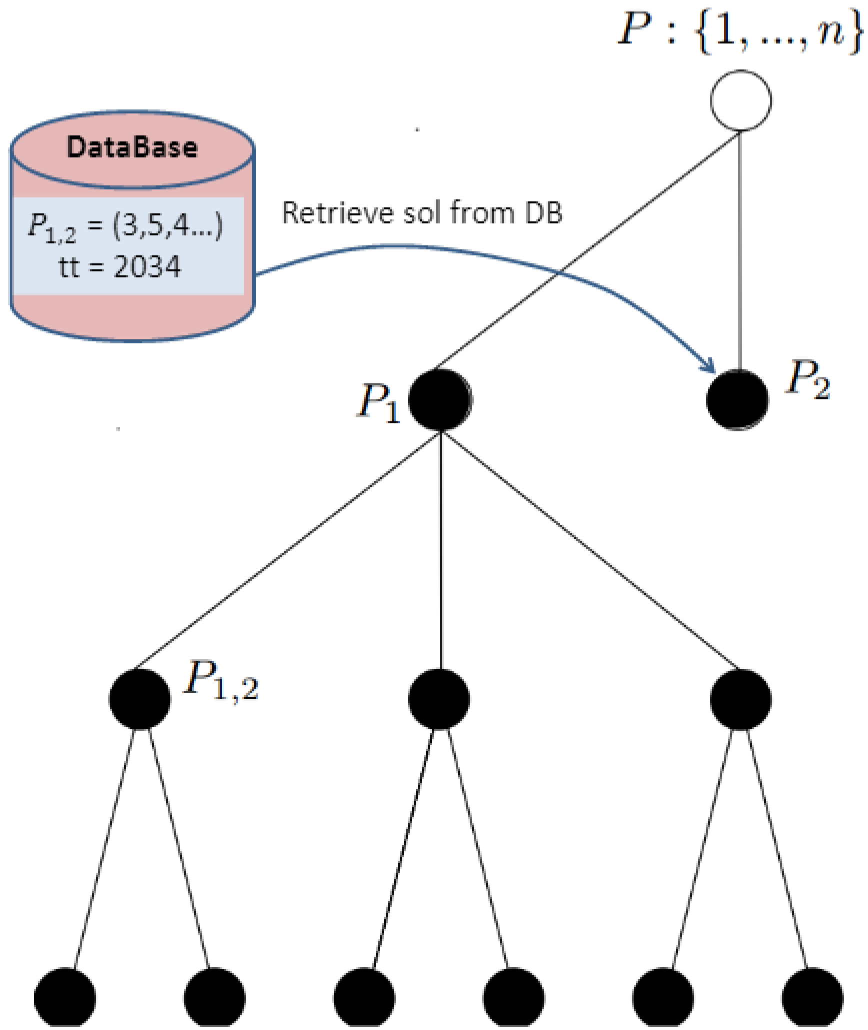
In Practice: Memorization



- « Never solve a problem twice »

The power of Memorization

- BB2001 of Szwarc et al. has no LB procedure:
  - Paradox (Szwarc et al. 2001): removal of LB evaluation drastically accelerate the solution.
  - => cut a sub-problem many times by computing LB is slower than solving it once and memorize the solution

- Can be further boosted!

Enhanced Paradox

- Enhanced Paradox (our work)
  - Removing **Split** from BB2001 drastically accelerate the solution
  - **Split** : decompose the problem by precedence relations

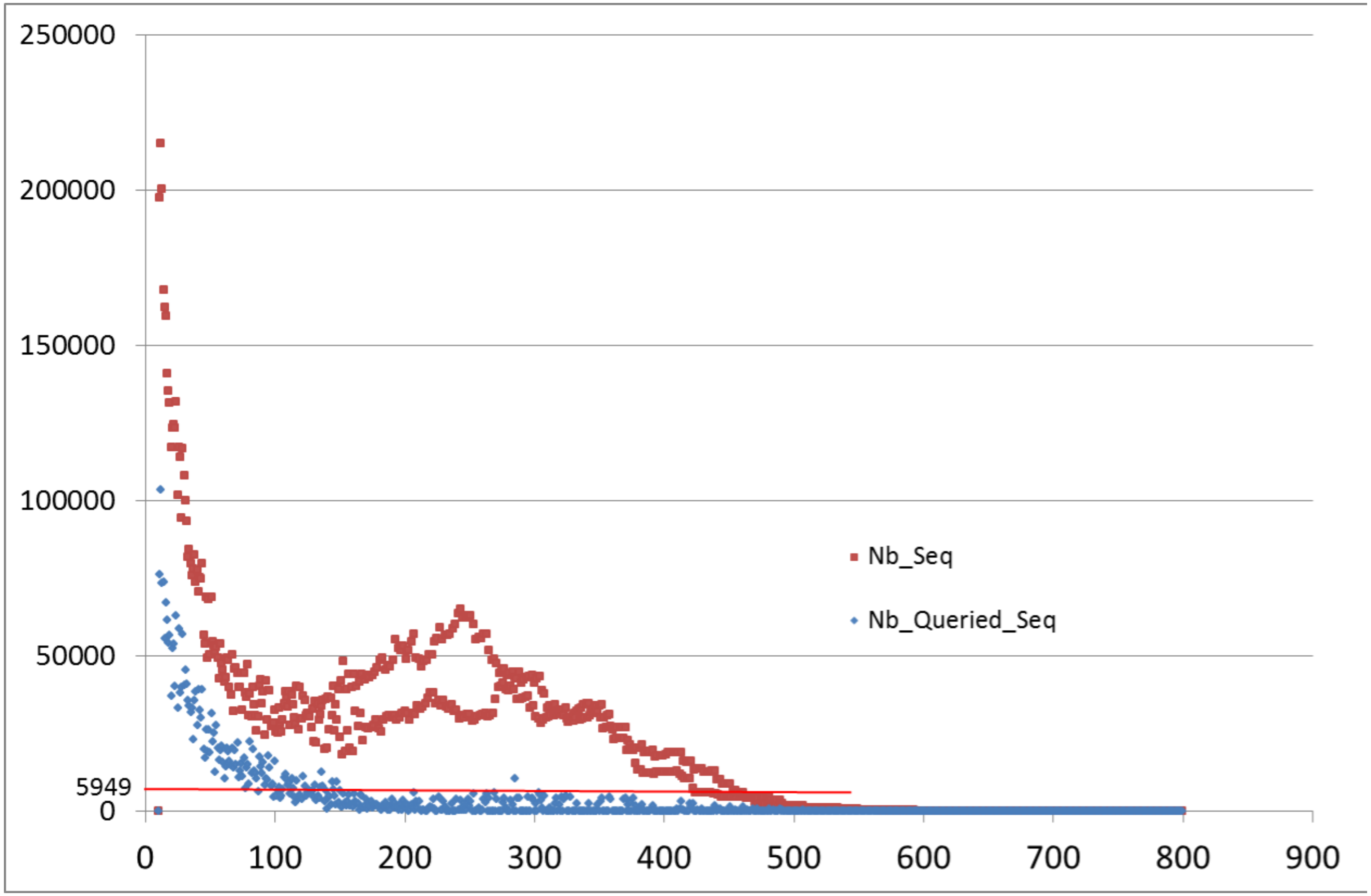
TMin (s)	TAvg (s)	TMax (s)	#Nodes	#Hit	#SolMem
0.0	192.81	2963.0	880268	227203	111175
0.0	8.0	114.0	3053648	899031	1262895

Table: Results for instances of size 700

- But…the memory is filled quickly (solve up to 700 jobs)

Memory Analysis

- Are all memorized solutions useful ?



Memory Analysis

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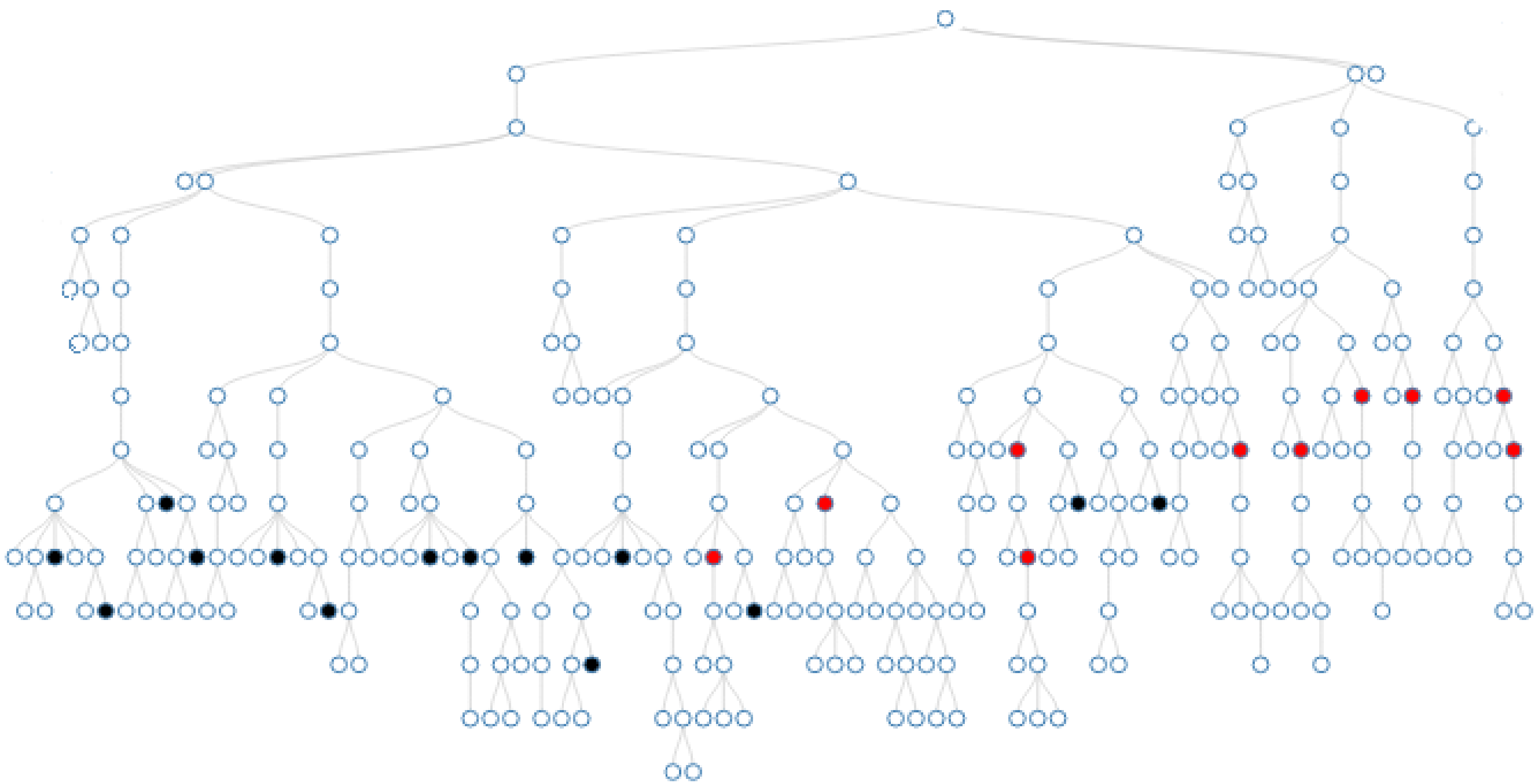

Memory Cleaning Strategies

- LUFO (Least Used First Out)
  - Attach a counter (`nbUsed`) to each solution
  - When a solution is used: `nbUsed=nbUsed+1`
  - Memory full: `nbUsed=nbUsed-1` for all solution, remove a solution if its `nbUsed<0`
- Also tested:
  - FIFO (First In First Out)
  - BEFO (Biggest Entry First Out)

	TMin	TAvg	TMax	#Nodes	SizeMem
FIFO-800	0.0	60.0	3144.0	16161758	1727397
BEFO-800	0.0	59.0	4828.0	6356245	2006948
LUFO-800	0.0	19.0	275.0	5408511	1354477
LUFO-1200	0.0	192.0	3763.0	28223765	1424612

Summary 2

- An enhanced paradox for $1||\sum T_i$
- An efficient memory cleaning strategy: LUFO
- Solve instances with up to 1200 jobs (from 900)
- Work done together with:
  - Federico Della Croce
  - Vincent T'Kindt

Further: a Branch & Memorize framework

- We have witnessed the power of Memorization
- Can be applied on other problems?
- Three problems are considered: $1|r_i|\sum C_i$, $1\left|\tilde{d}\right|\sum w_i C_i$, $F2||\sum C_i$
- Treated in T'Kindt et al. (2004).
  - Different search strategies are revisited
  - The so-called DP property is implemented
    - Consider two nodes: 123{4,..,n} vs 132{4,..,n}
    - If 123 dominates 132, then the second node should be cut
    - Use memory to store the prefixed part

Further: a Branch & Memorize framework

A framework: different ways of doing Memorization:

- Solution Memorization ($1||\sum T_i$)
  - Depth-first

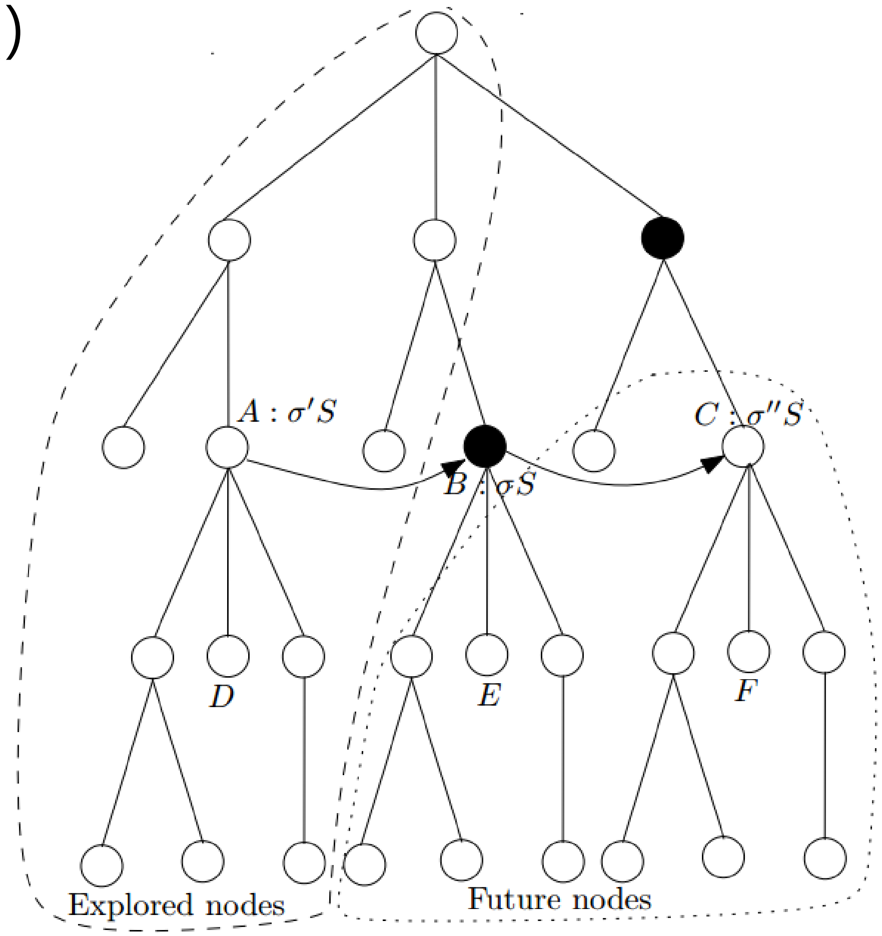

Figure 1: Solution Memorization

Further: a Branch & Memorize framework

A framework: different ways of doing Memorization:

- Passive Node Memorization
  - Memorize the current best solution for the fixed part given by branching
  - Used for cutting
  - Consider σ' dominates σ and σ'' (breadth-first)

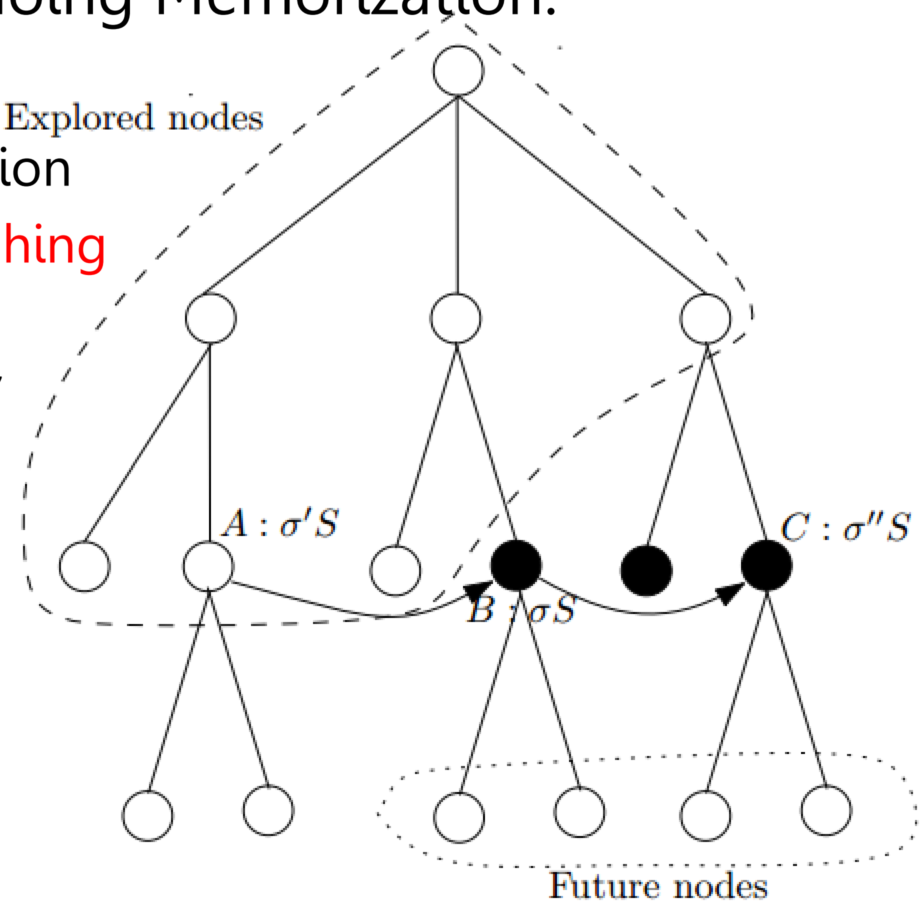

Figure 2: Passive node memorization)

Further: a Branch & Memorize framework

Different ways of doing Memorization:

- Predictive Node Memorization
  - Memorize the current best solution for the fixed part given by active search
  - Passive Node Memo + Local search
    - Dominance Rules Relying on Scheduled Jobs (Jouglet et al. 2004)
  - Used for cutting
  - Consider π dominates σ and σ''

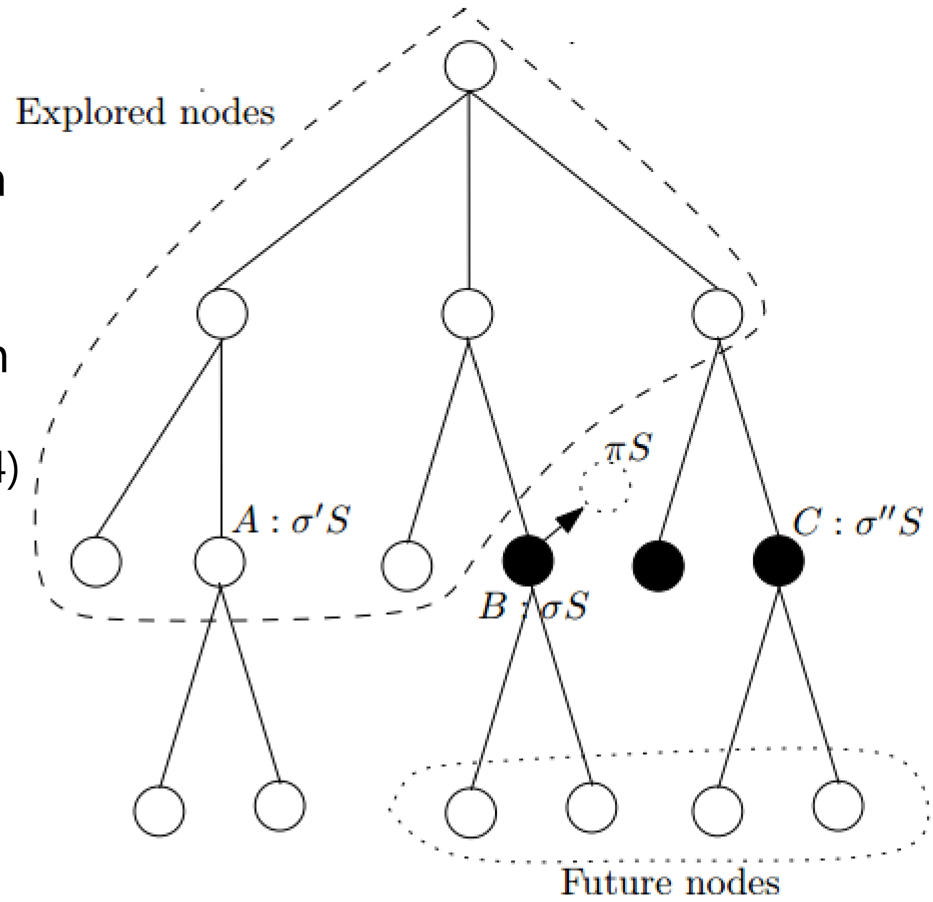

Figure 3: Predictive node memorization

Choose the right Memo scheme

Given a branching algorithm, choose a Memorization scheme

- Branching scheme
- Search strategy
- Other properties: whether « Decomposable »…

Choose the right Memo scheme

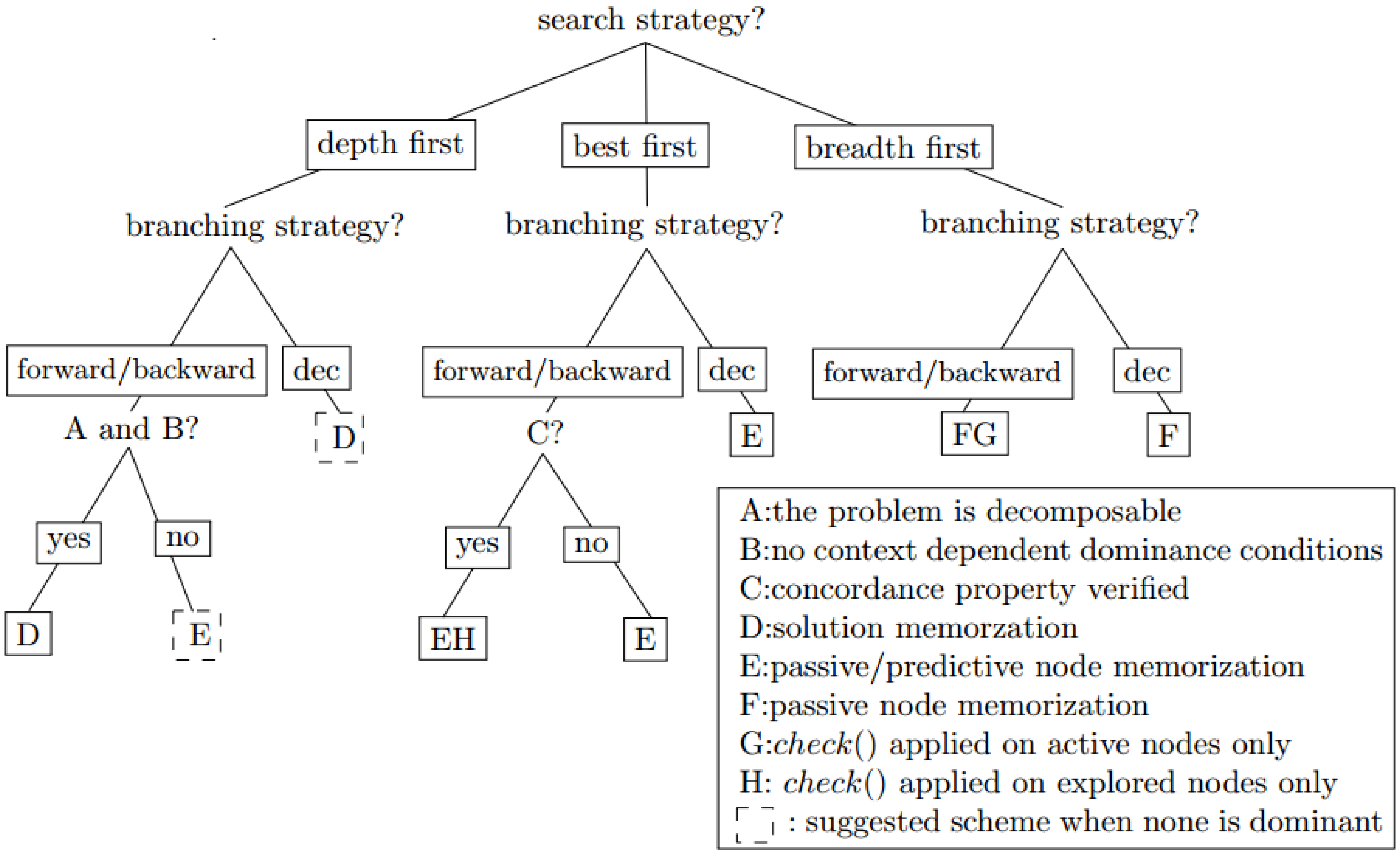

Figure 4: Decision tree for choosing the memorization scheme

Further: a Branch & Memorize framework

- The evidence of the power of memorization

Problem	Largest instances solved		Features of the best algorithm with memorization	Best in literature?
	Without memorization	With memorization		
$1\|r_i\|\sum C_i$	80 jobs	130 jobs	depth first+ *predictive node memorization*	yes
$1\|\tilde{d}_i\|\sum w_iC_i$	40 jobs	130 jobs	breadth first+ *passive node memorization*	yes
$F2\|\|\sum C_i$	30 jobs	40 jobs	best first+ *passive node memorization*	no
$1\|\|\sum T_i$	300 jobs	1200 jobs	depth first+ *solution memorization*	yes

Conclusion

- Part 3: work done together with:
  - Federico Della Croce
  - Vincent T'Kindt

- For theoretical guarantee: branch smartly and Merge !
- For practical efficiency: Branch & Memorize
  - Memorization is a powerful technique for scheduling problems
  - Should be considered as an essential building block of branching algorithms
  - The choice of branching scheme and search strategy are important


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
BIYAN
SHUKRIA
TINGKI
YAQHANYELAY
TASHAKKUR ATU
SUKSAMA
EKHMET
GRAZIE
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MAAKE
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