Mixed Time Frameworks for the Periodically Aggregated Resource-Constrained Project Scheduling Problem

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Journées GOThA/Bermudes 2017

École Polytechnique de l'Université de Tours

Tuesday 26th September, 2017

- Linear formulations for resource-constrained scheduling problems
  - ILP formulations based on time-indexed variables
  - Introduction of continuous time variables (event-based)

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  - Precise handling of activity start and completion times

C. Artigues, M. Gendreau, L.-M. Rousseau, A. Vergnaud.
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In: Computers & Operations Research 36 (8) pp. 2330–2340, 2009.

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- RCPSP/π: more general, purely discrete problem

J. Böttcher, A. Drexl, R. Kolish, F. Salewski.
Project Scheduling Under Partially Renewable Resource Constraints.
In: Management Science 45 (4) pp. 543–559, 1999.

Input

- $\mathcal{A}$ Set of n activities
- $\mathcal{R}$ Set of m renewable resources
- p_i Processing time of activity $i \in \mathcal{A}$
- b_k Capacity of resource $k \in \mathcal{R}$
- $r_{i,k}$ Request of activity $i \in \mathcal{A}$ on resource $k \in \mathcal{R}$
- E $\subset \mathcal{A} \times \mathcal{A}$; end-to-start precedence relations
- Δ Period length

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Constraints

1. Precedence constraints
2. Periodically aggregated resource constraints

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Objective

Minimise the project duration

Solution representation

S_i Start date of activity $i \in \mathcal{A}$

$d_{i,\ell}$ Length of $[(\ell - 1)\Delta, \ell\Delta] \cap [S_i, S_i + p_i]$

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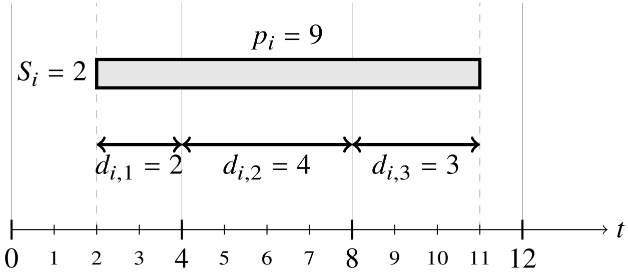

Abstract formulation

$$\begin{aligned} \text{Minimise} \quad & S_{n+1} - S_0 & \\ \text{s.t.} \quad & S_{i_2} - S_{i_1} \geq p_{i_1} & \forall (i_1, i_2) \in E \\ & \sum_{i \in \mathcal{A}} r_{i,k} \frac{d_{i,\ell}}{\Delta} \leq b_k & \forall k \in \mathcal{R} \,,\ \forall \ell \in \mathbb{Z} \end{aligned}$$

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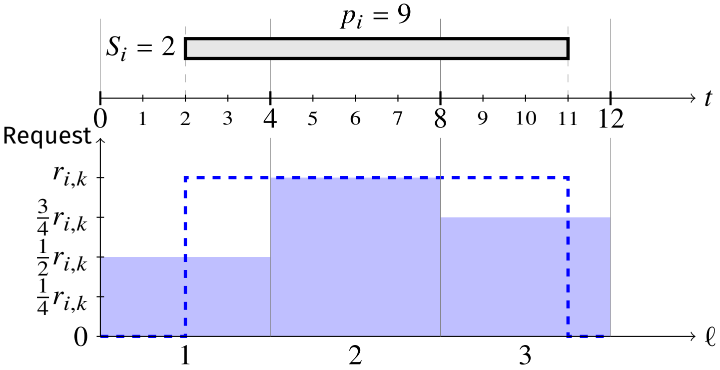

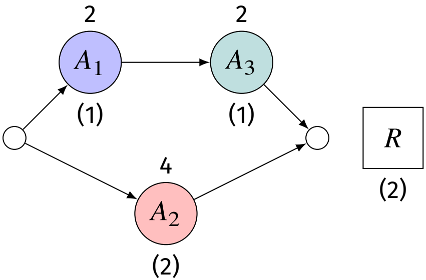
2
A1
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A3
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A2
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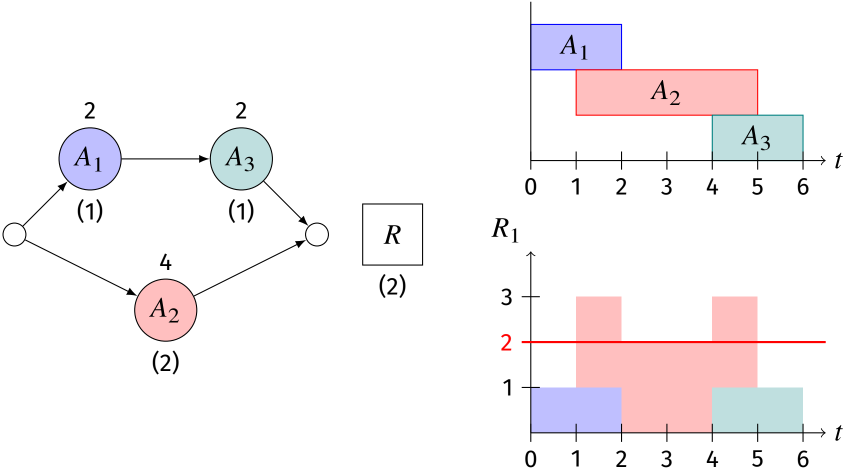
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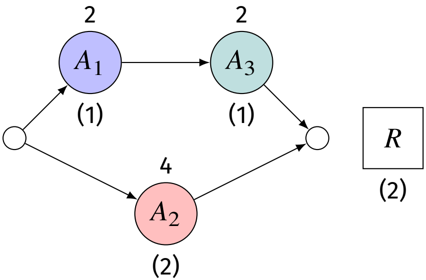
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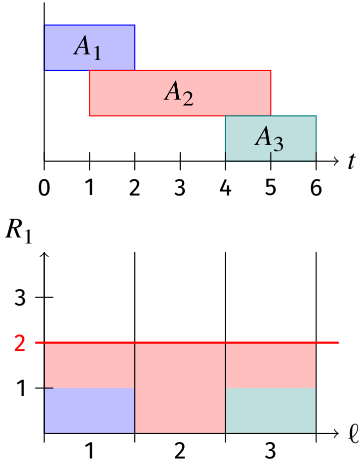
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R_1
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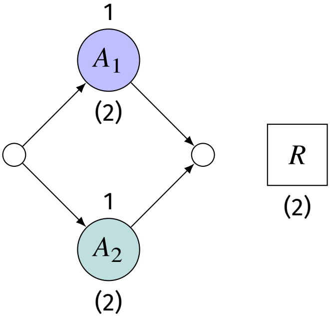
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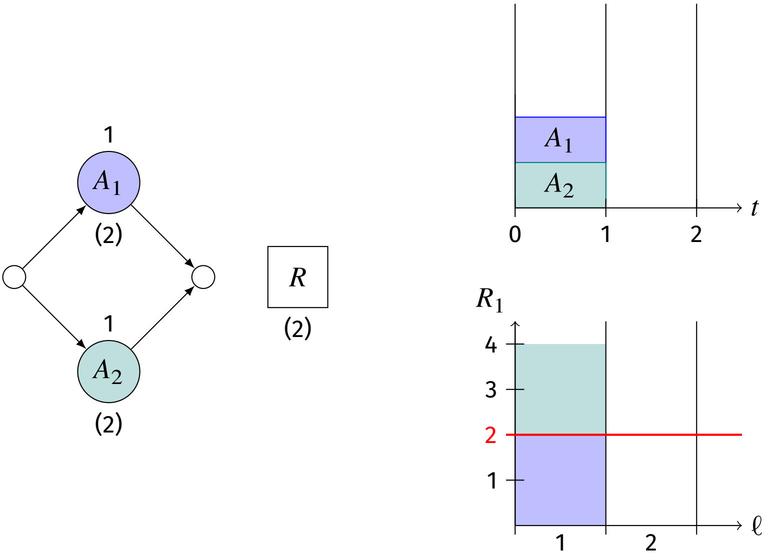
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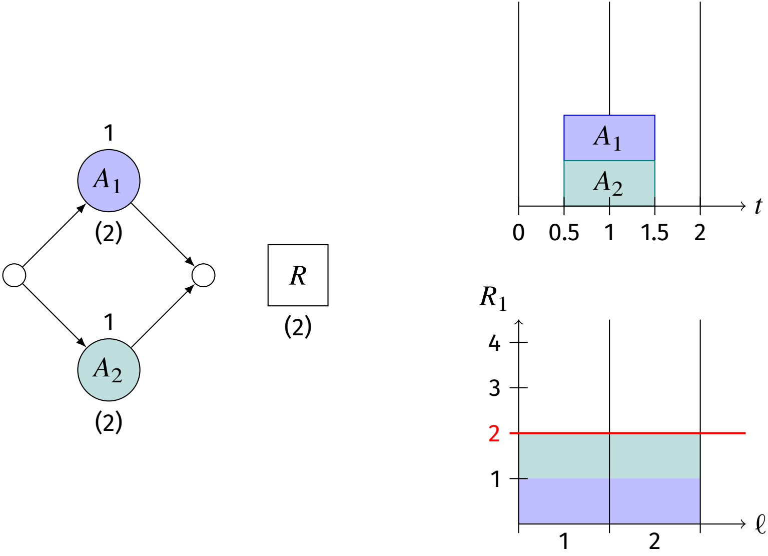
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A2
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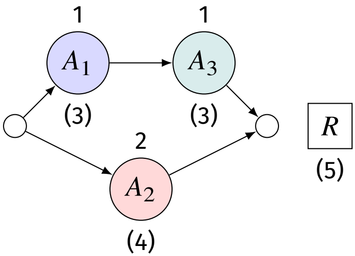
1
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A_3
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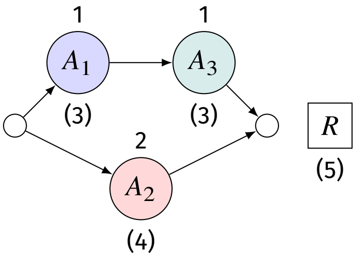
2
A_2
(4)
R
(5)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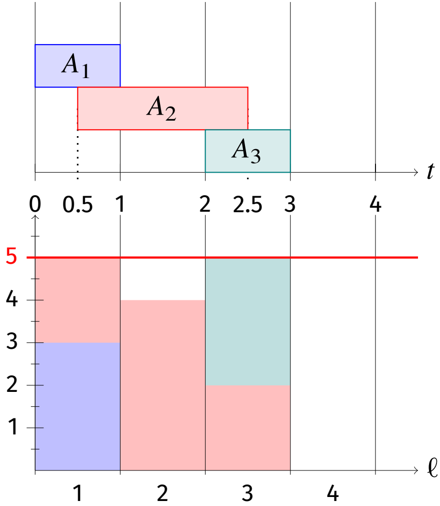
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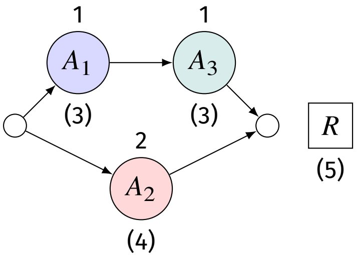
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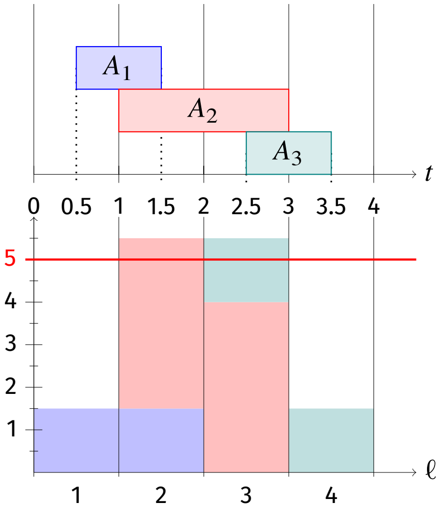
A_1
A_2
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Key points

- PARCPSP = RCPSP relaxation
- Substantial project duration reduction (even with small values of Δ)
- Shifting a schedule may alter its resource-feasibility (even with constant resource capacities over time)
- $0 \leq S_0 < \Delta \quad \Rightarrow \quad S_{n+1} - S_0 \leq S_{n+1} = C_{\max}$

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P.-A. Morin, C. Artigues, A. Haït, T. Kis, F. Spieksma.
Structural Properties and Complexity of the PARCPSP.
Technical report, LAAS-CNRS (University of Toulouse, France), 2017.

Theorem

The PARCPSP is NP-complete.

Additional data for models

- $\mathscr{L}$ Set of L consecutive aggregated periods (length Δ) starting from $t = 0$

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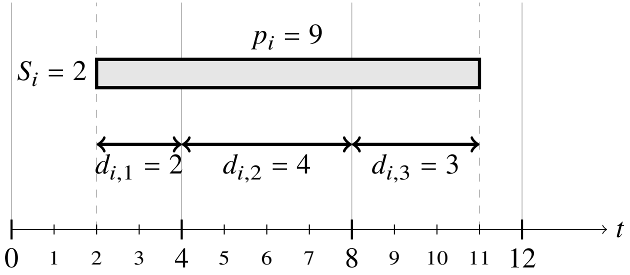
Mixed time representation management



How to link S_i and $d_{i,\ell}$?

First mixed time framework (MTF-1)

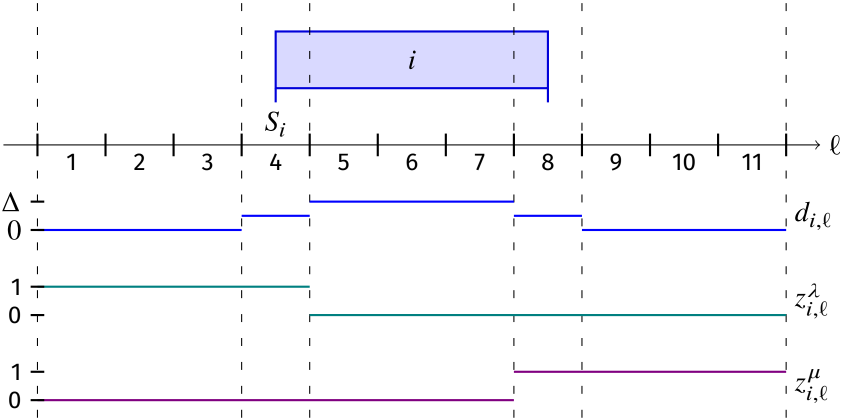
Variables

S_i Start date of activity $i \in \mathcal{A}$

$d_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i, S_i + p_i]$

$z^{\lambda}_{i,\ell}$ $= 1$ if activity $i \in \mathcal{A}$ starts in period $\ell \in \mathcal{L}$ or after, 0 otherwise

$z^{\mu}_{i,\ell}$ $= 1$ if activity $i \in \mathcal{A}$ completes in period $\ell \in \mathcal{L}$ or before, 0 otherwise



i
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Δ
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d_{i,ℓ}
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z^λ_{i,ℓ}
1
0
z^μ_{i,ℓ}

Step behavior of $z_{i,\ell}^{\lambda}$ and $z_{i,\ell}^{\mu}$

$$z_{i,\ell+1}^{\lambda} \leq z_{i,\ell}^{\lambda} \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$z_{i,1}^{\lambda} = 1 \qquad \forall i \in \mathscr{A}$$

$$z_{i,\ell-1}^{\mu} \leq z_{i,\ell}^{\mu} \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$z_{i,L}^{\mu} = 1 \qquad \forall i \in \mathscr{A}$$

Step behavior of $z^{\lambda}_{i,\ell}$ and $z^{\mu}_{i,\ell}$

$$z^{\lambda}_{i,\ell+1} \leq z^{\lambda}_{i,\ell} \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$z^{\lambda}_{i,1} = 1 \qquad \forall i \in \mathscr{A}$$

$$z^{\mu}_{i,\ell-1} \leq z^{\mu}_{i,\ell} \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$z^{\mu}_{i,L} = 1 \qquad \forall i \in \mathscr{A}$$

Bounding S_i

$$S_i \leq (\ell - 1)\Delta + \mathscr{M}\, z^{\lambda}_{i,\ell} \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$S_i \geq (\ell - 1)\Delta - \mathscr{M}(1 - z^{\lambda}_{i,\ell}) \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$S_i + p_i \leq \ell\Delta + \mathscr{M}(1 - z^{\mu}_{i,\ell}) \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

$$S_i + p_i \geq \ell\Delta - \mathscr{M}\, z^{\mu}_{i,\ell} \qquad \forall i \in \mathscr{A}\ ,\ \forall \ell \in \mathscr{L}$$

Computing $d_{i,\ell}$

$$d_{i,\ell} \geq \Delta(1 - z^{\lambda}_{i,\ell} - z^{\mu}_{i,\ell}) \qquad \forall i \in \mathscr{A}, \forall \ell \in \mathscr{L}$$

$$d_{i,\ell} \leq \Delta(1 - z^{\lambda}_{i,\ell+1} - z^{\mu}_{i,\ell-1}) \qquad \forall i \in \mathscr{A}, \forall \ell \in \mathscr{L}$$

$$d_{i,\ell} \geq (\ell\Delta - S_i) - \Delta\, z^{\mu}_{i,\ell} - \mathscr{M}(1 - z^{\lambda}_{i,\ell}) \qquad \forall i \in \mathscr{A}, \forall \ell \in \mathscr{L}$$

$$d_{i,\ell} \geq (S_i + p_i - (\ell - 1)\Delta) - \Delta\, z^{\lambda}_{i,\ell} - \mathscr{M}(1 - z^{\mu}_{i,\ell}) \qquad \forall i \in \mathscr{A}, \forall \ell \in \mathscr{L}$$

$$\sum_{\ell \in \mathscr{L}} d_{i,\ell} = p_i \qquad \forall i \in \mathscr{A}$$

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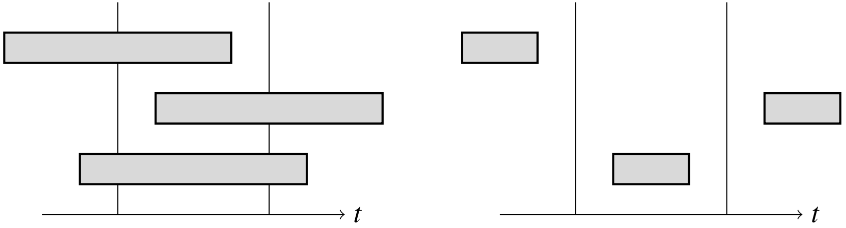


Second mixed time framework (MTF-2)

Variables

S_i Start date of activity $i \in \mathcal{A}$

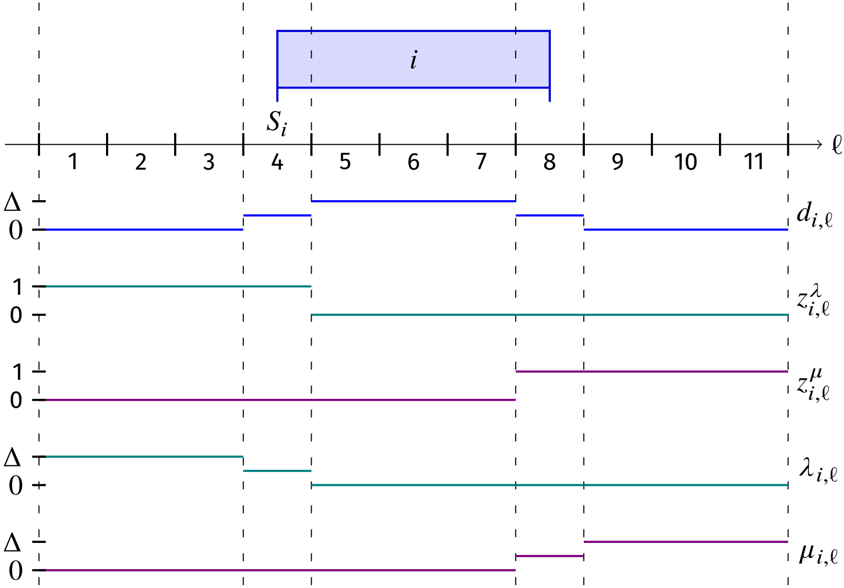
$d_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i, S_i + p_i]$

$z^{\lambda}_{i,\ell}$ $= 1$ if activity $i \in \mathcal{A}$ starts in period $\ell \in \mathcal{L}$ or after, 0 otherwise

$z^{\mu}_{i,\ell}$ $= 1$ if activity $i \in \mathcal{A}$ completes in period $\ell \in \mathcal{L}$ or before, 0 otherwise

$\lambda_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [0, S_i]$

$\mu_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i + p_i, L\Delta]$



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$d_{i,\ell}$
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$z^{\lambda}_{i,\ell}$
1
0
$z^{\mu}_{i,\ell}$
Δ
0
$\lambda_{i,\ell}$
Δ
0
$\mu_{i,\ell}$

Period partition

$$\lambda_{i,\ell} + d_{i,\ell} + \mu_{i,\ell} = \Delta \qquad \forall i \in \mathscr{A} \,,\ \forall \ell \in \mathscr{L}$$

Period partition

$$\lambda_{i,\ell} + d_{i,\ell} + \mu_{i,\ell} = \Delta \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

Interdependent step behavior

$$z^{\lambda}_{i,\ell+1} \leq \frac{\lambda_{i,\ell}}{\Delta} \leq z^{\lambda}_{i,\ell} \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

$$z^{\lambda}_{i,1} = 1 \qquad \forall i \in \mathcal{A}$$

$$z^{\mu}_{i,\ell-1} \leq \frac{\mu_{i,\ell}}{\Delta} \leq z^{\mu}_{i,\ell} \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

$$z^{\mu}_{i,L} = 1 \qquad \forall i \in \mathcal{A}$$

Computing S_i and $d_{i,\ell}$

$$S_i = \sum_{\ell \in \mathcal{A}} \lambda_{i,\ell} \qquad \forall i \in \mathcal{A}$$

$$\sum_{\ell \in \mathcal{A}} d_{i,\ell} = p_i \qquad \forall i \in \mathcal{A}$$

$$d_{i,\ell} \geq 0 \qquad \forall i \in \mathcal{A}, \forall \ell \in \mathcal{L}$$

Computing S_i and $d_{i,\ell}$

$$S_i = \sum_{\ell \in \mathcal{A}} \lambda_{i,\ell} \qquad \forall i \in \mathcal{A}$$

$$\sum_{\ell \in \mathcal{A}} d_{i,\ell} = p_i \qquad \forall i \in \mathcal{A}$$

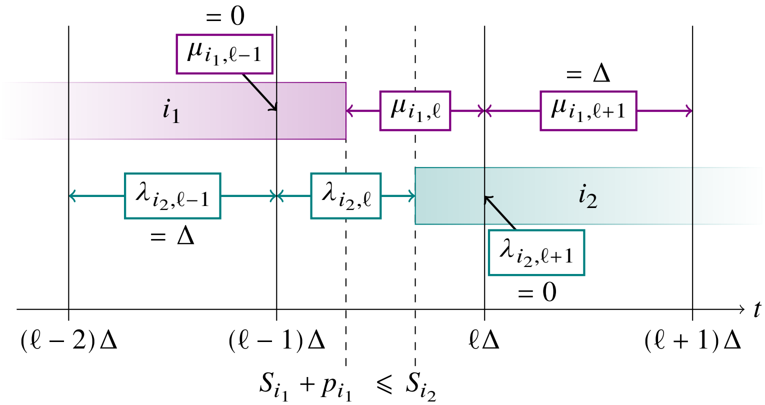
$$d_{i,\ell} \geq 0 \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

Note

$$S_i + p_i = \sum_{\ell \in \mathcal{A}} \lambda_{i,\ell} + \sum_{\ell \in \mathcal{A}} d_{i,\ell} = L\Delta - \sum_{\ell \in \mathcal{A}} \mu_{i,\ell} \qquad \forall i \in \mathcal{A}$$

Disaggregation of precedence constraints

$$\mu_{i_1,\ell} + \lambda_{i_2,\ell} \geq \Delta \qquad \forall (i_1, i_2) \in E\,,\ \forall \ell \in \mathscr{L}$$



Model sizes

Formulation	(MTF-1)	(MTF-2)
# variables	$n + 3nL$	$n + 5nL$
# binary variables	$2nL$	
# common constraints	$\lvert E\rvert + mL$	
# specific constraints	$n + 11nL$	$2n + 6nL$
# specific constraints with $\mathscr{M}$	$6nL$	0

Theorem

$\mathscr{X}_j$ *Solution space for (MTF-j)*

$\overline{\mathscr{F}}_j$ *Feasible solution space for the linear relaxation of (MTF-j)*

$$\text{Proj}_{\mathscr{X}_1}\left(\overline{\mathscr{F}}_2\right) \subseteq \overline{\mathscr{F}}_1$$

Theorem

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$\overline{\mathscr{F}}_j$ *Feasible solution space for the linear relaxation of (MTF-j)*

$$\mathrm{Proj}_{\mathscr{X}_1}\left(\overline{\mathscr{F}}_2\right) \subseteq \overline{\mathscr{F}}_1$$

Proof (partial)

$$\begin{aligned} d_{i,\ell} - \Delta\left(1 - z^{\lambda}_{i,\ell} - z^{\mu}_{i,\ell}\right) &= \Delta - \lambda_{i,\ell} - \mu_{i,\ell} - \Delta + \Delta z^{\lambda}_{i,\ell} + \Delta z^{\mu}_{i,\ell} \\ &= \Delta\left(z^{\lambda}_{i,\ell} - \frac{\lambda_{i,\ell}}{\Delta}\right) + \Delta\left(z^{\mu}_{i,\ell} - \frac{\mu_{i,\ell}}{\Delta}\right) \\ &\geq 0 \end{aligned}$$

$$\begin{aligned} d_{i,\ell} - \Delta\left(1 - z^{\lambda}_{i,\ell+1} - z^{\mu}_{i,\ell-1}\right) &= \Delta - \lambda_{i,\ell} - \mu_{i,\ell} - \Delta + \Delta z^{\lambda}_{i,\ell+1} + \Delta z^{\mu}_{i,\ell-1} \\ &= \Delta\left(z^{\lambda}_{i,\ell+1} - \frac{\lambda_{i,\ell}}{\Delta}\right) + \Delta\left(z^{\mu}_{i,\ell-1} - \frac{\mu_{i,\ell}}{\Delta}\right) \\ &\leq 0 \end{aligned}$$

Experiments

- PSPLIB – J30 instances

Δ	LB_1^*/LB_2^*	LB_1/LB_2	UB_1/UB_2	$t_1(s)$	$t_2(s)$
5	0.9143	0.9971	0.9937	159	161
4	0.8806	0.9946	1.0198	210	205
3	0.8520	0.9963	1.0261	242	241
2	0.8081	0.9892	1.0238	301	298
1	0.7769	0.9986	0.9945	565	571

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Δ	LB_1^*/LB_2^*	LB_1/LB_2	UB_1/UB_2	$t_1(s)$	$t_2(s)$
5	0.9143	0.9971	0.9937	159	161
4	0.8806	0.9946	1.0198	210	205
3	0.8520	0.9963	1.0261	242	241
2	0.8081	0.9892	1.0238	301	298
1	0.7769	0.9986	0.9945	565	571

Theorem

Formulation (MTF-2) is stronger than (MTF-1).

Periodically Aggregated Resource-Constrained Project Scheduling Problem

- Exact models based on mixed time frameworks
- Improvement of the linear relaxation

Periodically Aggregated Resource-Constrained Project Scheduling Problem

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Perspectives

- Modelling: consolidation of the link between continuous time and time-indexed variables
- Adaptation of energetic reasonning
- Integration into a planning/scheduling scheme

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[1]Department DISC, ISAE Supaéro, University of Toulouse, Toulouse, France

[2]Team ROC, LAAS CNRS, University of Toulouse, CNRS, Toulouse, France

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École Polytechnique de l'Université de Tours

Tuesday 26th September, 2017

Institut Supérieur de l'Aéronautique et de l'Espace SUPAERO

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